Commonsense notions of proximity and direction in environmental space *

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Abstract
It is desirable that formal theories of qualitative reasoning should be informed by the ways in which humans conceptualize the spaces in which they live. The work described here uses data provided in experiments with human subjects to derive some regularities in such conceptualizations. Two experiments with human subjects concerning the vague spatial relations ‘near’ and ‘left’ in environmental space are described. The method is to approach the boundary between truth and falsity of vague concepts by dividing the subject population into two groups and to ask separate questions about the truth or falsity of the existence of specific spatial relations to the separate groups. Thus, individuals are not required to face the boundary cases explicitly. The results are analyzed using several approaches and regularities considered, in particular the relationship between human conceptualization of the space and ‘ground truth’.

Introduction
Qualitative spatial reasoning is concerned with representing and reasoning with our commonsense knowledge of the spatial aspects of the physical world (Cohn & Hazarika 2001). There is a need for formal theories of spatial representation and reasoning to be cognitively plausible, and so properly guided by the way humans actually think about space. Proximity and orientation are two basic components of the ontology of space, and this paper describes experiments with human subjects on aspects of these categories. The spaces investigated are at the environmental scales, that is at the scale of “buildings, neighborhoods, and cities” (Montello 1993). Such spaces cannot be apprehended in a single viewing, and useful knowledge of them can be gained only by a series of observations over time and from different locations in the space. This sets them apart from ‘table-top’ spaces, where the space and its constituent entities can be more or less taken in at one observation (Zubin 1989).

The experiments concerned places on the campus of Keele University, UK (see figure 1), an area of 600 acres set within landscaped grounds. The area is well suited for the experiments, being neither abnormally flat or hilly, having an irregular network of paths, buildings and roads, having no highly dominant landmarks, and being of a size to be classified as an environmental space.

Some results on the first experiment on nearness have been reported in (Worboys 2001; Duckham & Worboys 2001). Since the publications of those papers, the work has moved on, and in this paper this paper we summarize the results from earlier, describe results of the direction experiment, and discuss some of the population-level characteristics of both. Overall, the results of this work show that while individual differences are important, there are striking regularities in the population’s notions of distance and direction in the space. We have also factored out individual differences of place, so that lines of visibility, obstacles, landmarks and paths are not considered in this work, except in a generalized sense. The experiments are qualitative in na-
tive, exploring subject’s views of ‘nearness’ and ‘leftness’ relationships between assemblages of places in the space. However, the analyses are often quantitative, making use of statistical techniques for significance testing and Dempster-Shafer Theory for analysis of degrees of belief.

**Literature background**

In addition to inherently qualitative work on topological relationships (Egenhofer & Franzosa 1991) and shape (Cohn 1995), there has also been some work on computational aspects of qualitative distance (Hernández, Clementini, & Di Felice 1995; Gahegan 1995) and qualitative direction (Ligozat 1993). Sadalla et al. (1980) observed asymmetries in the human perception of nearness, with more significant reference points or landmarks generally being understood to be near to adjacent points more frequently than vice versa. Stevens and Coupe (1978) and Hirtle and Jonides (1985) provide evidence of distortions in human spatial cognition resulting from the apparent hierarchical arrangement of places according to spatial and semantic criteria. Distortions such as asymmetry, vagueness, landmarks, hierarchies and clustering have proved difficult to integrate with the logical systems which form the backbone of computational approaches to qualitative spatial reasoning. Nevertheless, Tversky (1992) argues that these distortions are important cognitive devices that help humans to organize spatial information. Ideally, cognitively plausible computational models of qualitative spatial information should be able to allow for such distortions.

Some research has begun to provide a basis for closer integration between the computational and the cognitive. The ‘egg-yolk’ calculus (Cohn & Gots 1996) has proved useful as a formal framework for reasoning about vague or indeterminate boundaries. Robinson (1990; 2000) used an adaptive algorithm to produce a fuzzy membership function for nearness based on experiments with human subjects.

**The experiments**

Data was collected from human subjects in two structurally similar experiments, concerned with nearness and leftness respectively. The experiments are designed to elicit levels of support for spatial relationships between places in the space. As with most qualitative spatial relationships, nearness and leftness are vague, in the sense that determination of truth or falsity of the relationships may be unclear for some borderline cases. The form of the experiments, derived from ideas in (Bonini et al., 1999), is designed to determine degree of support indirectly.

The experiments were conducted during 2000–2001 on the Keele University campus, UK, where the authors were then employed (see figure 1). For the nearness experiment, a group of 22 human subjects were asked to complete a series of questionnaires concerning the nearness of places on the campus to each other. The just over twenty places were selected as being well known (‘significant’) places on Campus, identified using a preliminary study. Half the subjects (the truth group) were asked in a questionnaire to check on the list of significant places those places they thought it was true to say were near to a fixed reference place, drawn from the collection of significant places. The other half of the subjects (the falsity group) were asked to identify those places for which it was false to say were they near to the same reference place. With a break of at least one day between successive questionnaires, each subject was then asked to complete further questionnaires, one for each reference place, until information about all reference places had been gathered for each subject. All the subjects were Keele University staff with some years experience of the Campus and were asked to complete questionnaires without thinking too much and certainly without reference to maps. For each reference place, tallies were made of which places were checked by the two groups. For example, Table 1 shows the tallies for the case where the reference place is the Library. Full details of this experiment with some analysis of results is contained in (Worboys 2001; Duckham & Worboys 2001).

The leftness experiment was of the same form. In this case the Library was taken as fixed throughout the questionnaires. In each questionnaire, subjects were asked to imagine standing at one of the reference places facing the Library, and to identify each place for which it was true (false) to say that that place was on the left. As with the nearness experiment, each subject was asked to complete questionnaires, one for each reference place, until information about all reference places had been gathered for each subject. Vague directional predicates such as "left" and "right" are commonly found in the literature on spatial cognition (e.g., (Frank 1992)) and so were obvious choices for this

<table>
<thead>
<tr>
<th>Place</th>
<th>Truth group</th>
<th>Falsity group</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 hour Reception</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Academic Affairs</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Barnes Hall</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Biological Sciences</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Chancellors Building</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Chapel</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Chemistry</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Clock House</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Computer Science</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Earth Sciences</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Health Centre</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>Holly Cross</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>Horwood Hall</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Keele Hall</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Lakes</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>Leisure Centre</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Library</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Lindsay Hall</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Observatory</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Physics</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Students Union</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Visual Arts</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1: Nearness tallies for the Library
study. It was assumed that there would be no substantial differences between a study of leftness or rightness (aside from the reversal of direction), and so the decision to study “left” rather than “right” was an arbitrary one.

One important difference in the phrasing of the nearness and leftness questions needs to be highlighted. There is a common distinction in the literature between egocentric and allocentric frames of reference in spatial cognition (Hart & Moore 1973). The nearness experiment used an allocentric question, concerning the nearness of places independently of the respondent’s location or perspective. The leftness experiment used an egocentric question, which explicitly asked respondents to imagine standing at a particular location facing a particular direction. The reason for this change was that preliminary investigations suggested that allocentric forms of the leftness question were generally harder to understand than more natural egocentric versions. Allocentric questions about leftness are difficult to phrase clearly without introducing additional complexities, such as asking respondents imagine vectors connecting different reference places to be placed in a particular relationship to the reference place, providing a reference place and a list of all other places to be imagined in a particular direction. In a similar way, a \( \chi^2 \) (chi-squared) test was used to determine the significance of support for the relation \( \nu(p,q) \) provided by the data in arrays \( l_T(p,q) \) and \( l_D(p,q) \). This test is in the form of a tally of votes for the truth or falsity of the corresponding statements about \( \nu(p,q) \) and \( \lambda(p,q) \). The tallies are contained in four integer arrays,

\[
\begin{align*}
\{n_T(p,q)|p,q \in P\} \\
\{n_D(p,q)|p,q \in P\} \\
\{l_T(p,q)|p,q \in P\} \\
\{l_D(p,q)|p,q \in P\}
\end{align*}
\]

where the entries indicate the number of subjects voting for the truth/falsity of the appropriate nearness/leftness relation. Let the total number in each of the truth and falsity groups be \( G \).

**Analysis and discussion**

**Significance tests**

A \( \chi^2 \) (chi-squared) test was used to determine the significance of support for the relation \( \nu(p,q) \) provided by the data in arrays \( n_T(p,q) \) and \( n_D(p,q) \). So, many votes from the truth group and few for falsity group could indicate significant validation of the relation, whereas the reverse case could indicate significant validation of its negation. If a significance level \( s \) is fixed, then the analysis results in three possible truth values for \( \nu(p,q) \) and \( l_T(p,q) \):

- significantly the case, denoted \( \top \)
- significantly not the case, denoted \( \bot \)
- undetermined by the experiment, denoted \( ? \).

For ease of expression, we use formulations such as \( \nu(p,q) \equiv \top \) (or \( \nu(p,q) \equiv \top \) where no confusion arises) to indicate truth valuations induced by significance level \( s \).

The notion of ‘nearness’ is a similarity relation, where the formal properties of equivalence (reflexivity, symmetry and transitivity) are weakened. Nearness may be assumed to be reflexive by definition, but symmetry is not born out by this work. For example, when \( s = 0.001 \), the Chapel is judged to be definitely near to Academic Affairs, but Academic Affairs is not definitely near to the Chapel. However, using the three truth values, the results do exhibit weak symmetry. For all significance levels \( s \) less than 0.169, the following holds:

\[
\forall p,q \in P. \nu(p,q) = \top \rightarrow \nu(p,q) \neq \bot \tag{1}
\]

While as might be expected the relation is not transitive, it does also exhibit weak transitivity. For all significance levels \( s \) less than 0.002, the following holds:

\[
\forall p,q \in P. (\nu(p,q) = \top \land \nu(q,r) = \top) \rightarrow \nu(p,r) \neq \bot \tag{2}
\]

In a similar way, a \( \chi^2 \) (chi-squared) test was used to determine the significance of support for the relation \( \lambda(p,q) \) provided by the data in arrays \( l_T \) and \( l_D \). In this case, neither symmetry nor transitivity are to be expected. In fact we could expect a form of antisymmetry, because if when at \( p \) facing the library \( q \) is significantly to the left, it cannot be that when at \( q \) facing the library \( p \) is significantly to the left. This is born out by the data in the following way. For all significance levels \( s \):

\[
\forall p,q \in P. \lambda(p,q) \neq \top \rightarrow \lambda(q,p) = \top \tag{3}
\]

For all significance levels \( s \) less than 0.03, the following holds:

\[
\forall p,q \in P. \lambda(p,q) \neq \top \rightarrow \lambda(q,p) = \top \tag{4}
\]

**Support metrics**

For places \( p, q \in P \), support for the truth or falsity of \( n_T(p,q) \) and \( l_T(p,q) \) is given by the tallies in the arrays \( n_T \), \( n_D \), \( l_T \), and \( l_D \). The issue addressed here is how to measure this support. The most simple-minded approach is to take the difference between the tallies for the truth and falsity groups to generate support metrics \( \sigma_\nu, \sigma_\lambda \) as follows:

\[
\sigma_\nu(p,q) = \frac{n_T(p,q) - n_D(p,q)}{G} \tag{5}
\]

\[
\sigma_\lambda(p,q) = \frac{l_T(p,q) - l_D(p,q)}{G} \tag{6}
\]

Metrics \( \sigma_\nu \) and \( \sigma_\lambda \) lie between \(-1\) and \(1\), where a value of \(-1\) indicates complete support for the falsity of the relation,
zero indicates no support either way, and −1 indicates complete support for its truth.

Because in each experiment there is evidence both for and against the propositions, an alternative approach is to use Dempster’s rule of combination (Shafer 1976) to give support metrics. This gives the following metrics $\sigma_\nu^+$ and $\sigma_\chi^-$ for degree of support for the truth of the nearness and leftness relationships, respectively.

$$\sigma_\nu^+(p, q) = \frac{n_\nu^+(p, q)(G - n_\chi(p, q))}{G^2 - n_\nu^+(p, q)n_\nu(p, q)}$$

$$\sigma_\chi^-(p, q) = \frac{l_\nu(p, q)(G - l_\chi(p, q))}{G^2 - l_\nu(p, q)l_\nu(p, q)}$$

The metrics $\sigma_\nu^-$ and $\sigma_\chi^-$ for degree of support for the falsity of the nearness and leftness relationships, respectively, are given by:

$$\sigma_\nu^-(p, q) = \frac{n_\nu(p, q)(G - n_\chi(p, q))}{G^2 - n_\nu(p, q)n_\chi(p, q)}$$

$$\sigma_\chi^-(p, q) = \frac{l_\nu(p, q)(G - l_\chi(p, q))}{G^2 - l_\nu(p, q)l_\chi(p, q)}$$

This family of support functions lie between 0 and 1, where 0 indicates no support and 1 indicates full support for the appropriate proposition.

Commonsense geometry and the surveyed world

An interesting question concerns the relationship between peoples commonsense view of what is near what and left of what and the ‘objective’ world measured by surveyors and mapped by cartographers. In order to understand overall effects it is important to ‘normalize’ the data. In the case of the nearness experiment, each questionnaire provides very limited context to the respondent; a reference place, list of target places, and a question about the truth/falsity of the nearness relation between the reference place and each of the targets. However, the list of places does provide some context, and an initial question is whether this context influences the response. To see that it does have a partial effect, compare figures 2 and 3.

In figure 2, support $\sigma_\nu(p, q)$ is plotted against distance on the ground, $|p - q|$, and one sees an expected relationship where as distance increases so support for nearness decreases. In figure 3, the distance has been normalized to take account of the contexts provided by the lists of places, so for reference places on the periphery of the campus, where distances to targets is on average further than for central reference places, a compensating factor (based on the square root of the mean of squares of distances to the reference) has been applied. Figure 3 shows the ‘sharpening up’ of the resulting distribution. The graph of the Dempster support function $\sigma_\nu^+$ plotted against the normalized distances is shown in figure 4.

To see the overall structure of the leftness data, the positions of places has been normalized as follows. For places $p, q \in P$, the data pertains to the situation where a respondent is asked to evaluate truth/falsity of leftness of $p$ from the viewpoint of being positioned at $q$ and facing the Library. A coordinate frame is set up with the Library is fixed in position (1,0), and $q$ at the origin. The normalized position of $p$ is then given by its coordinates in this coordinate frame. Figure 5 shows the result of plotting support $\sigma_\chi(p, q)$ against the angular component (in degrees) of the normalized position of $p$ for all $p, q \in P$.

Figure 5 strikingly shows the regularities in correspondence between human perception of spatial relationships, and those relationships as surveyed on the ground and presented in conventional maps. The overall shape of the ‘curve’ is not unlike the familiar sine wave for the trigonometry of Euclidean space. It may be noticed that there are some significant errors in perception of angular relations in a few cases. Three places at an angle of between 120° and 135° have been voted not left. This reflects people’s perceptions of positions on the campus. Special cases such as this need further investigation for topographic interferences such as curved roads, hills, and landmarks.

Two further features may be pointed out. The first is the slight lack of symmetry about the vertical axis. The intercept with the horizontal axis occurs at between about 30° and 15°. This reflects the slight lack of symmetry in our un-
understanding of the leftness relation. If something is directly ahead of us, it is not just that we are undecided about its leftness, but we are likely to consider it not to our left. The second feature is the maximum of the curve occurs at an angle less than 90°. This indicates a more surprising feature of human understanding of the leftness relation, where we feel most strongly that something is to the left of us if it is almost perpendicular to the direction of view and slightly in front. This property, and a corresponding property for the minimum, deserve further analysis.

Another visualization of these features is shown in figures 6 and 7, where Dempster’s support metrics $\sigma^{+}_{\lambda}(p, q)$ and $\sigma^{-}_{\lambda}(p, q)$ are plotted against the same normalized angle. The advantage of this visualization is that the two separate support metrics, measuring evidence for and against the leftness relation, enable some further insight. As can be seen, supporting evidence for leftness increases sharply around 0°, but is not zero at 0°. However, evidence against leftness at 0° is substantial at around the 0.8 level. So, it is not that there is no commitment either way about leftness at 0°, but that there is commitment both for and against, with the weight of evidence very much against. We can also see from the graphs that there is some support for leftness in directions behind the viewer at angles in the approximate range $-135^\circ$ to $-180^\circ$ (that is, in directions to the right and behind of the viewer). This phenomena needs further analysis, but could be because humans have more difficulty determining directions for places behind their mind’s eye field of view.

**Conclusions**

This paper has reported results from two experiments with human subjects concerning conceptions of direction and angle in environmental space. The paper has demonstrated that although there are differences between individual subjects’ views, and results vary from place to place, depending on the relevant topography, there are striking regularities. The experiments are limited in the number of subjects involved and the amount of information gleaned. Because of the required gap between questionnaires, subjects need to be committed over several weeks to respond to the questions, and there is a limit as to what can be expected. Also, the work has involved a single test site.

From the beginnings of geometry, much of the quantitative work on spatial relationships is founded on a notion of distance. A motivation for the work reported here was a need to have quantitatively richer theories of vagueness in the case of spatial relationships, and from preliminary work nearness seemed a more natural concept to work with than
distance. So the question remains what is the relationship between nearness and distance. ‘Near’ is clearly not the same as ‘not far’, and ‘far’ is not equivalent to ‘not near’, so no simple relationship obtains. Nearness is in the class of similarity concepts, and work from cognitive scientists on the inverse relationship between similarity and conceptual distance (e.g. (Gärdenfors 2000)) may throw some useful light here.

There are many promising further directions. The authors have already repeated the nearness experiment using the same site with different subjects, for validation purposes. The results of this are still being analyzed. There are plans to repeat the nearness and direction experiments at different sites; with the similar characteristics as the Keele Campus, but also different in scale, dimension (in-building spaces, for example), and including virtual spaces, such as the Web. With regard to direction, much more data needs to be obtained. This paper described the situation where all directions were towards the same reference place (the Library). This was required for reasons of practicality in limiting the size of the experiment. However, if a richer geometry of human conception of the spaces that we occupy is to emerge, more evidence is needed.

References


