Some more thoughts on change

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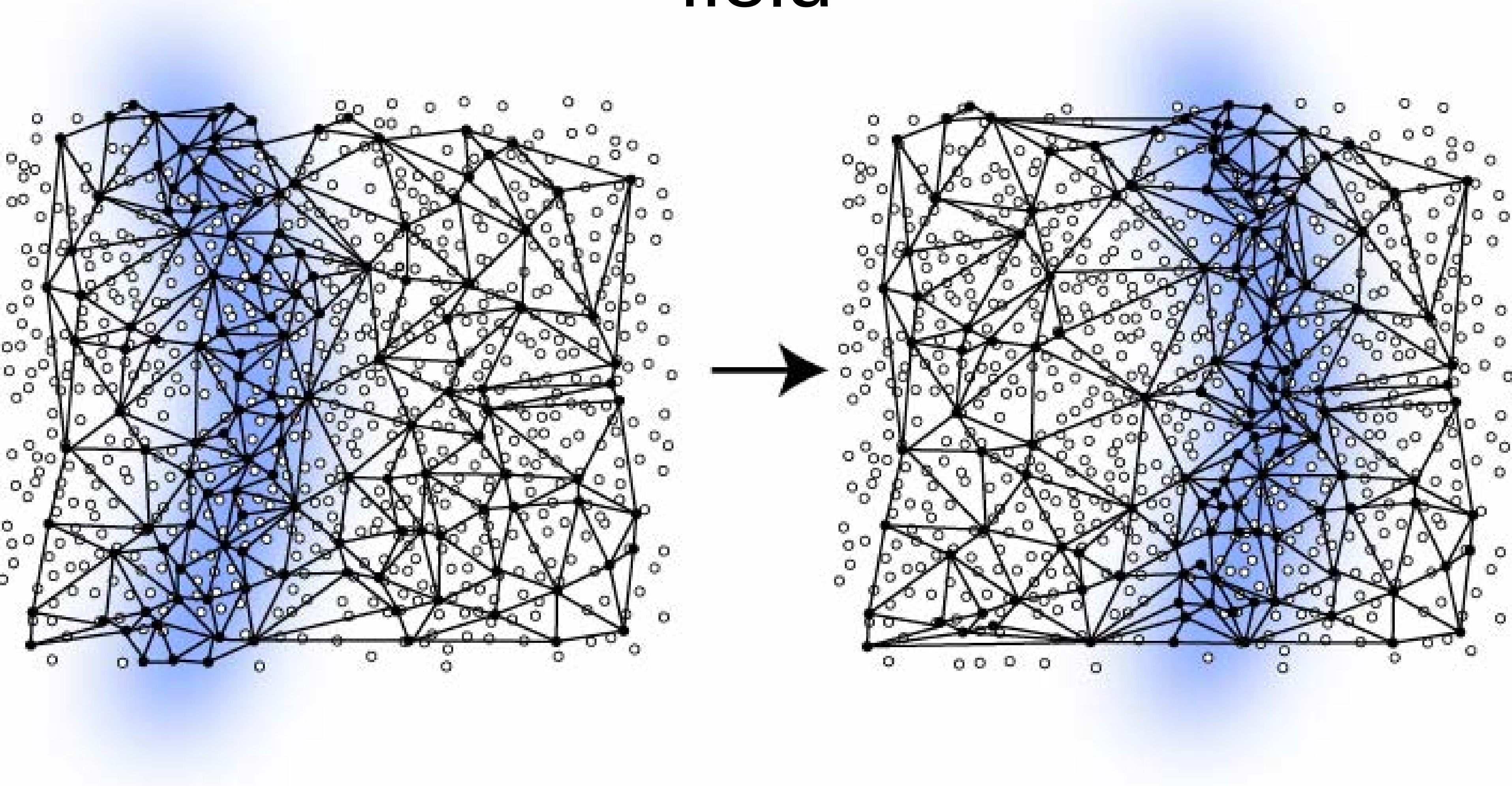
Sensors responding to a dynamic field



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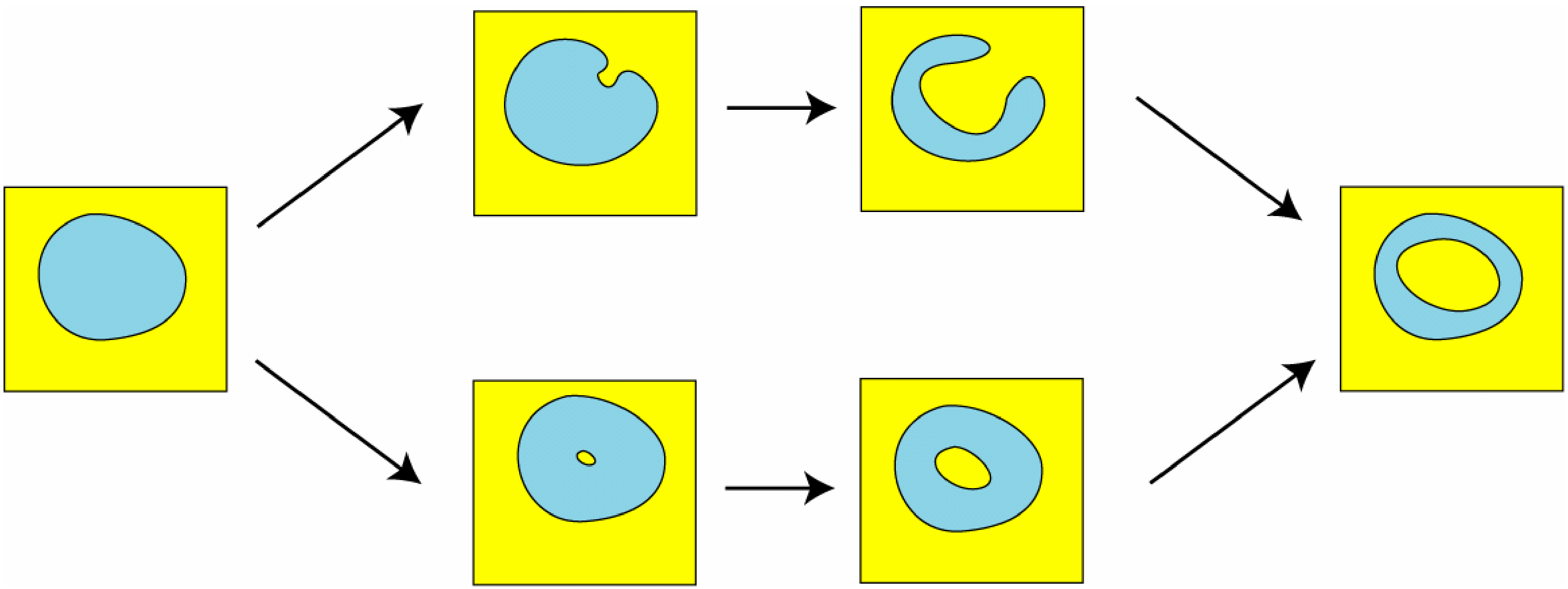
Topological change

Process not product:
Two topological changes



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Seeking the atoms

What are the “points, lines and polygons” of topological change

Some assumptions

- We are considering changes in spatial fields.
- The spatial framework is an infinite discretized Euclidean plane
- The discretization is such that we make no distinction between cells adjacent at a point or edge.
- The measurement domain is Boolean
- The temporal dimension is sufficiently fine-grained that only a single spatial cell can change at any one time instant (so, over a finite period of time, changes only occur in a finite area of the plane)
- We are interested in topological changes

What are the atoms?

- We are concerned then with topological changes of areal objects (areas where the Boolean measurement is 1, say).
- Can we give a finite list of types of topological change, from which all can be built?

A completeness result

- The only possible topological changes, based on the earlier assumptions are combinations of:
  - insert – delete
  - merge – split
  - self merge – self split

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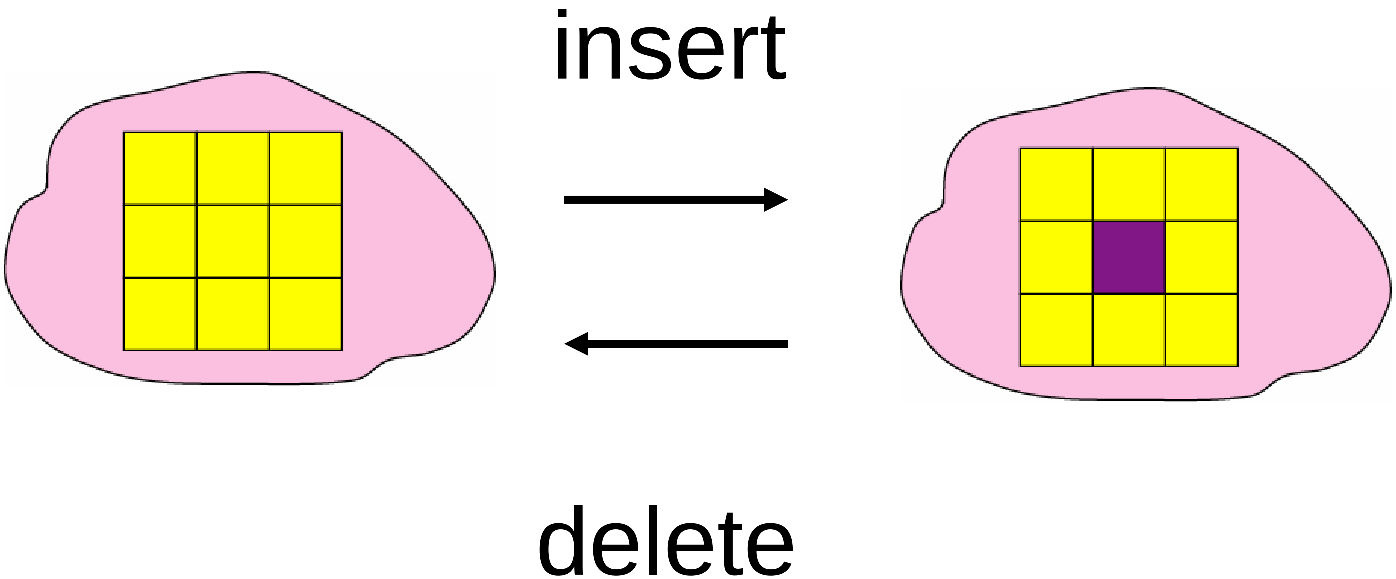
Proof sketch

- Consider the effect that a single cell in inducing a topological change.
- Assume a square grid for simplicity
- Catalog the possibilities



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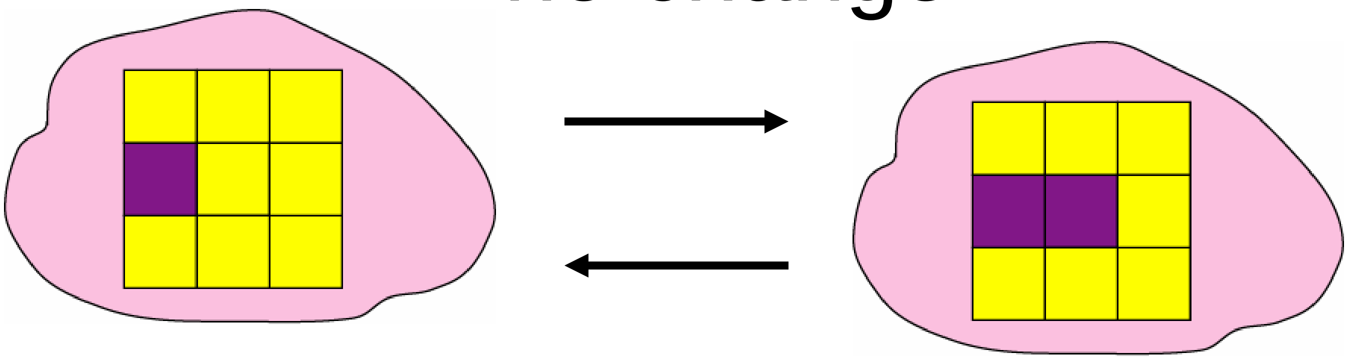
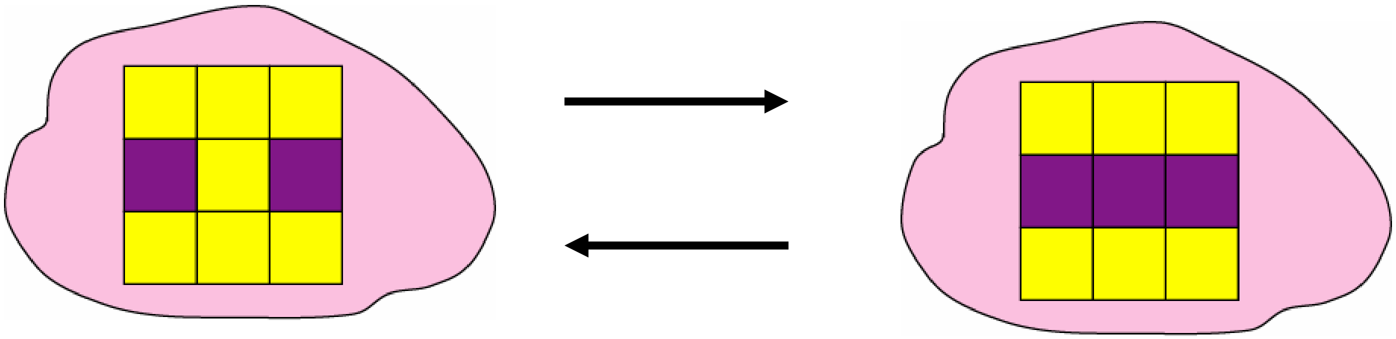
no change



no change

merge
self-merge



split
self-split

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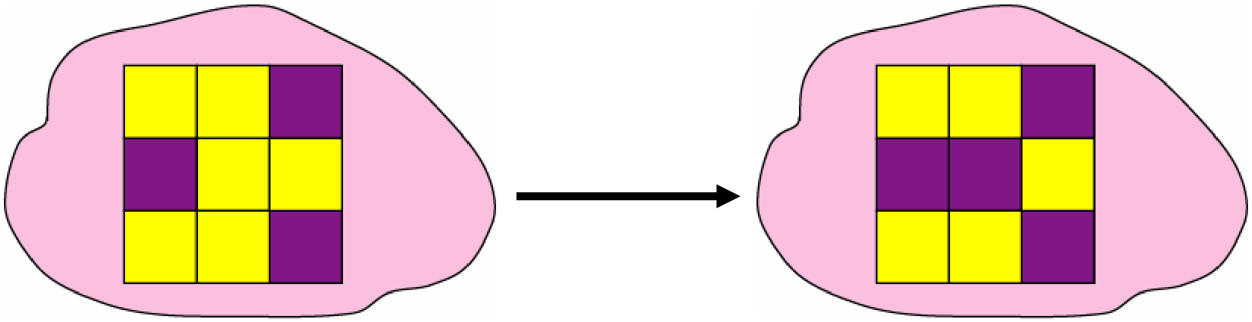
- … and that’s all we have, except that we can have more complex splits and merges.
- Example:



but this can be decomposed into two (self)-merges

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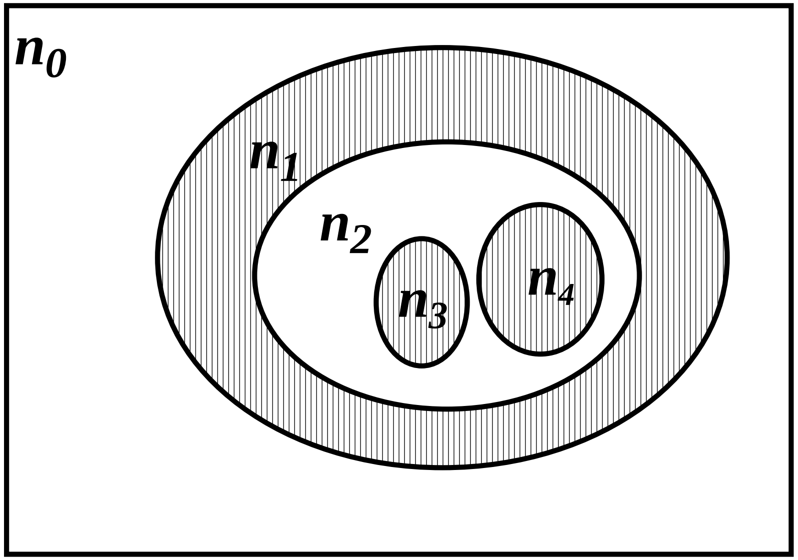
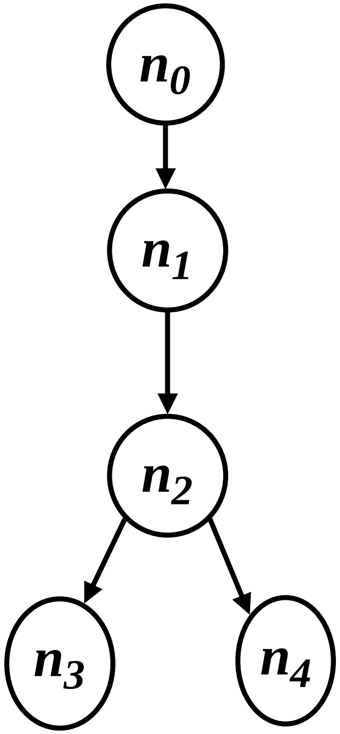
Directed Tree Model

Nodes — regions and holes

Directed edges — proper-part-of relations

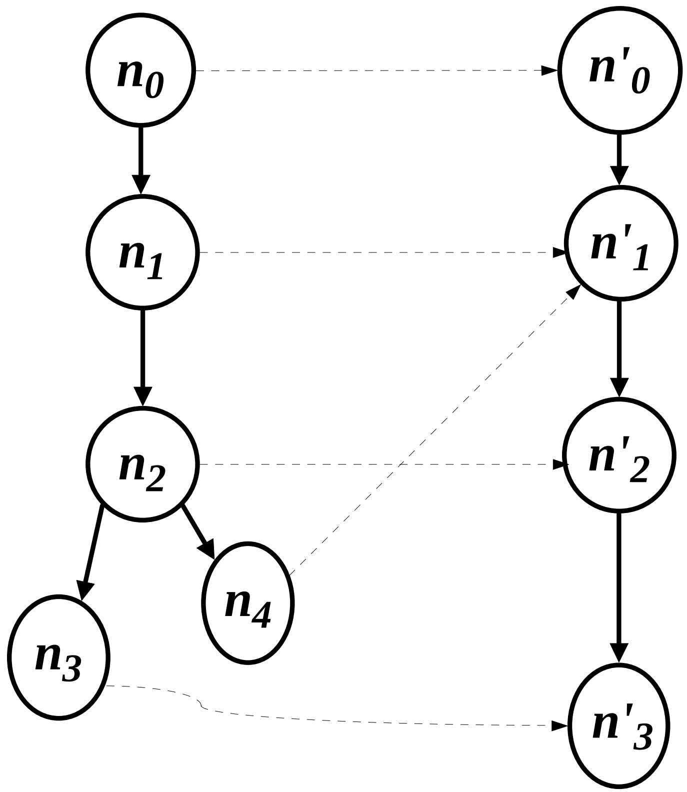




Specifying atomic changes: Tree morphisms

Tree morphism

- undirected graph morphism between trees
- maps root to root



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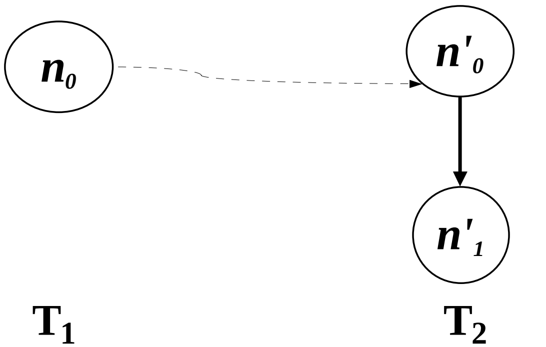
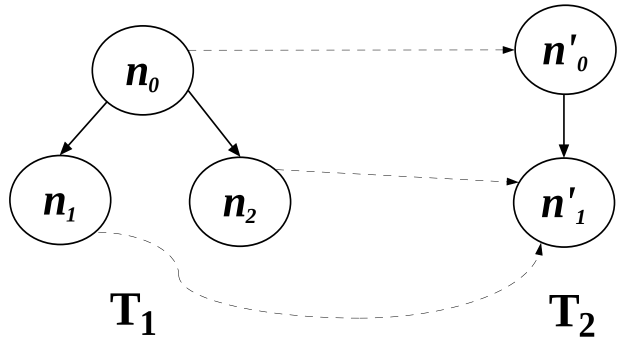
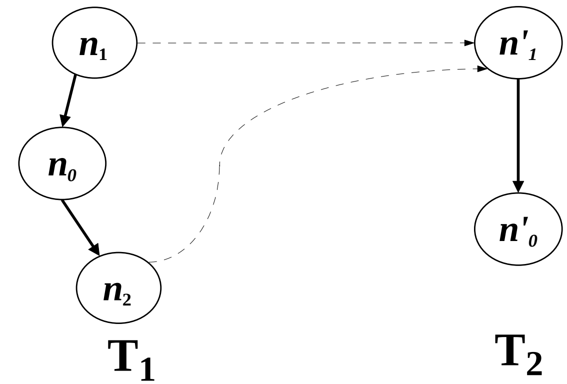
Atomic Changes



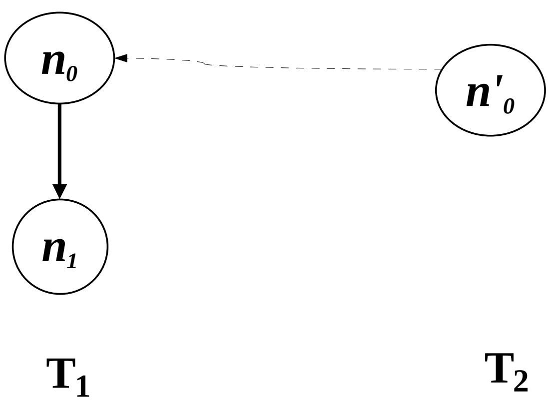
atomic insert



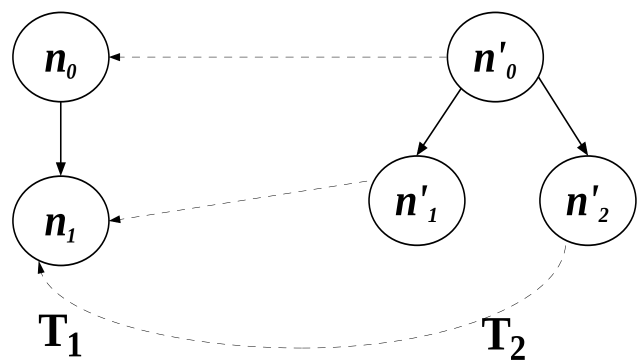
atomic merge I



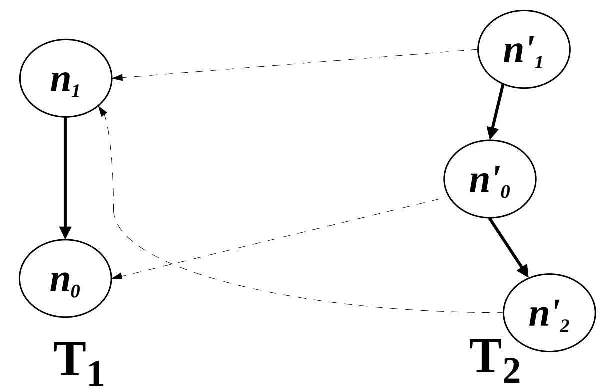
atomic merge II



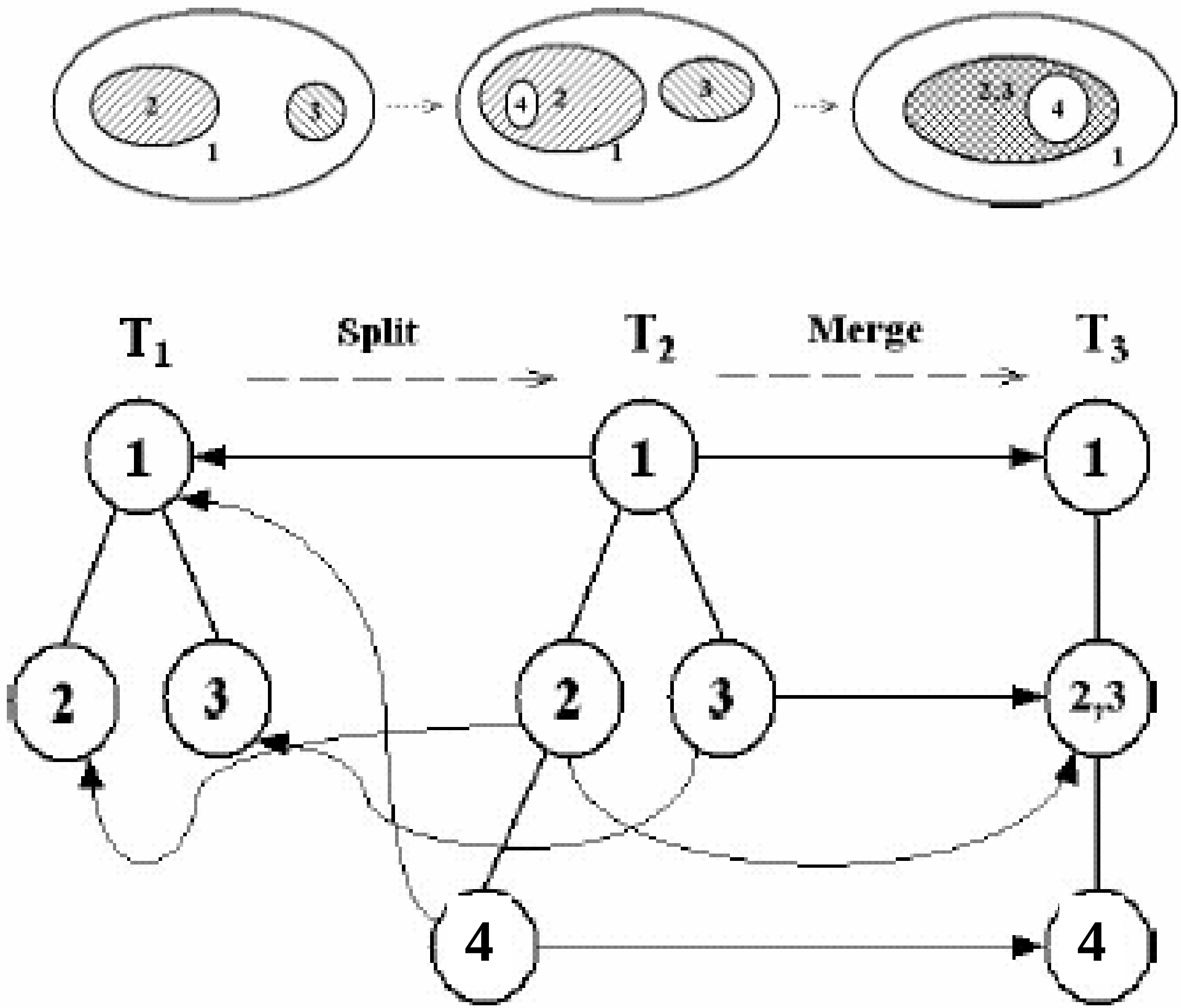
atomic delete



atomic split I



atomic split II



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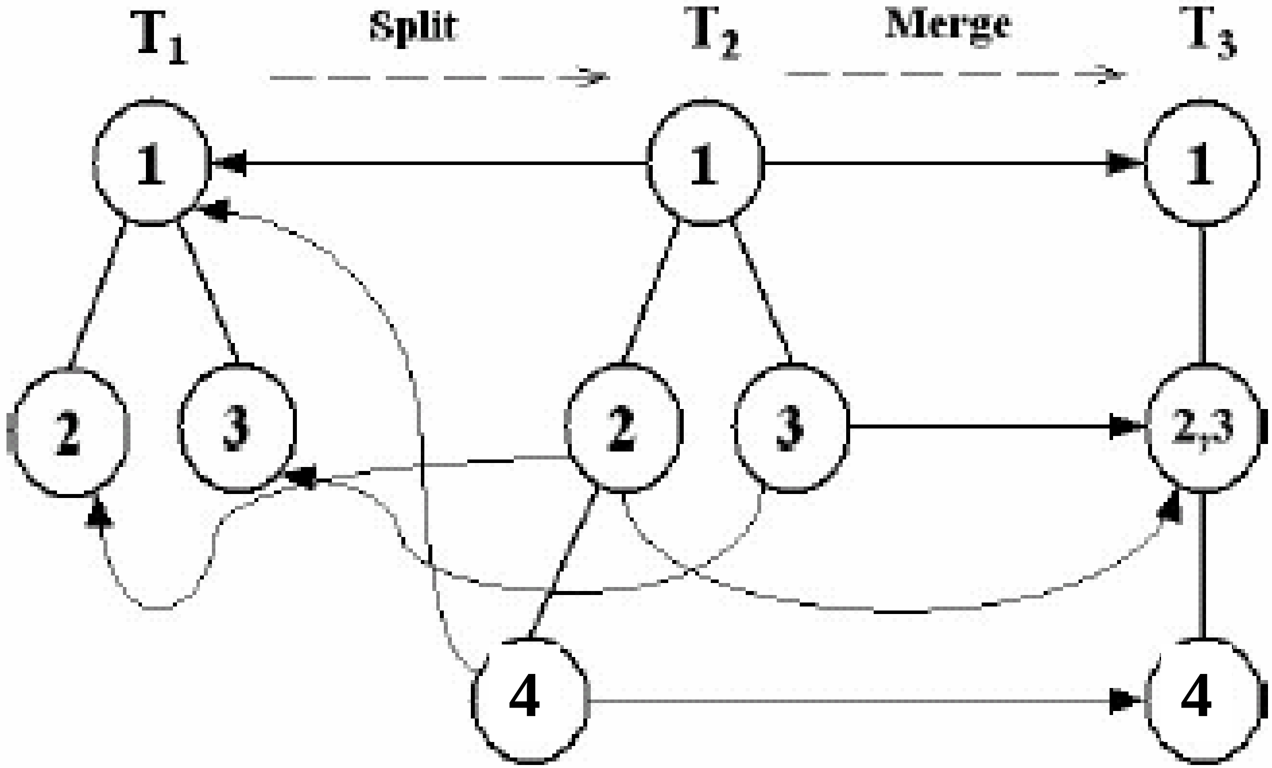

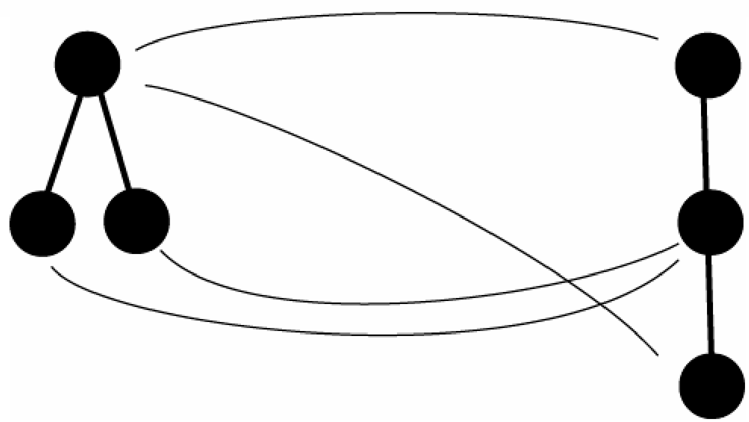

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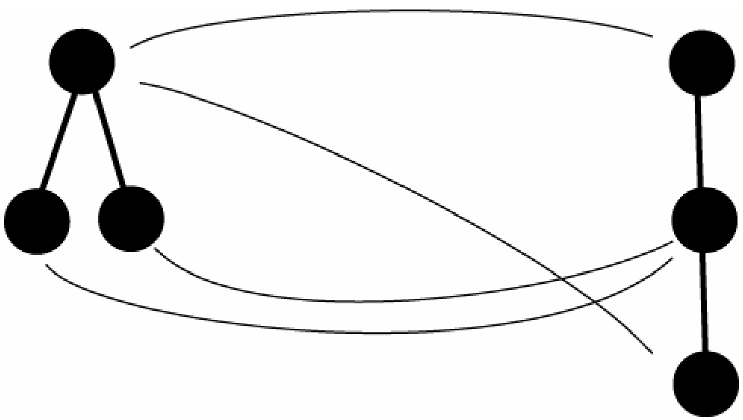
Relations on trees and their composition



- Such relations always send the root to the root, and preserve edges.
- We note that the depth-parity of nodes is preserved. This is a necessary condition. Is it, along with a root condition, sufficient?
- Open question: How can we characterize such relations?

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Specifying Complex Changes
The uniqueness (ISMD) Theorem

Every complex change is equivalent to a complex change of the form:

$$T_0 \overset{\gamma_0}{\dashrightarrow} T_1 \overset{\gamma_1}{\dashrightarrow} T_2 \overset{\gamma_2}{\dashrightarrow} T_3 \overset{\gamma_3}{\dashrightarrow} T_4$$

γ_0 is basic **I**nsert or no change

γ_1 is basic **S**plit or no change

γ_2 is basic **M**erge or no change

γ_3 is basic **D**elete or no change

… and moreover

- no other arrangement of I,S,D, and M, or subsets of I, S, D, and M gives all possible changes.

Another thought on ISMD

- I S | M D
- $(MD)^{-1} = D^{-1} M^{-1} = IS$
- Just look at SM.
- There is a general result about relations that parallels this.

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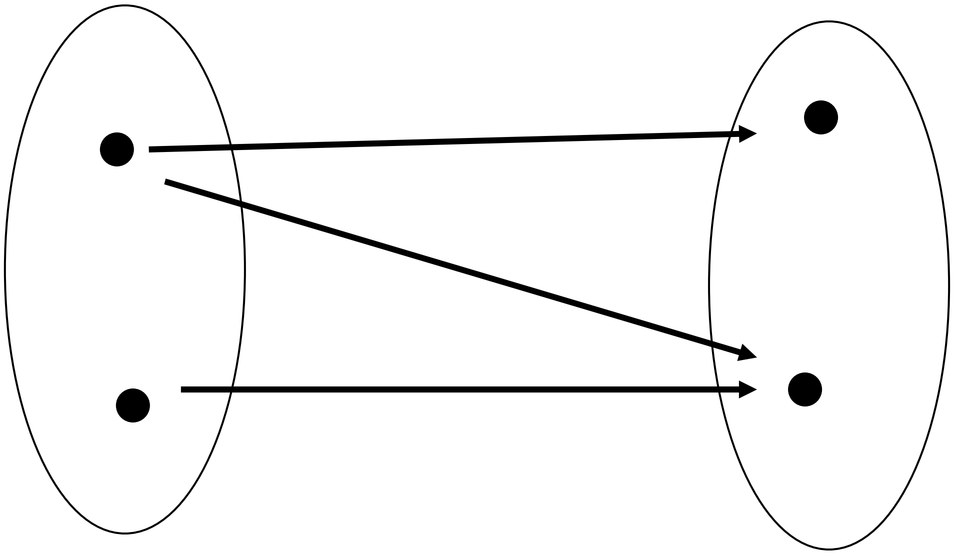

A total surjective relation

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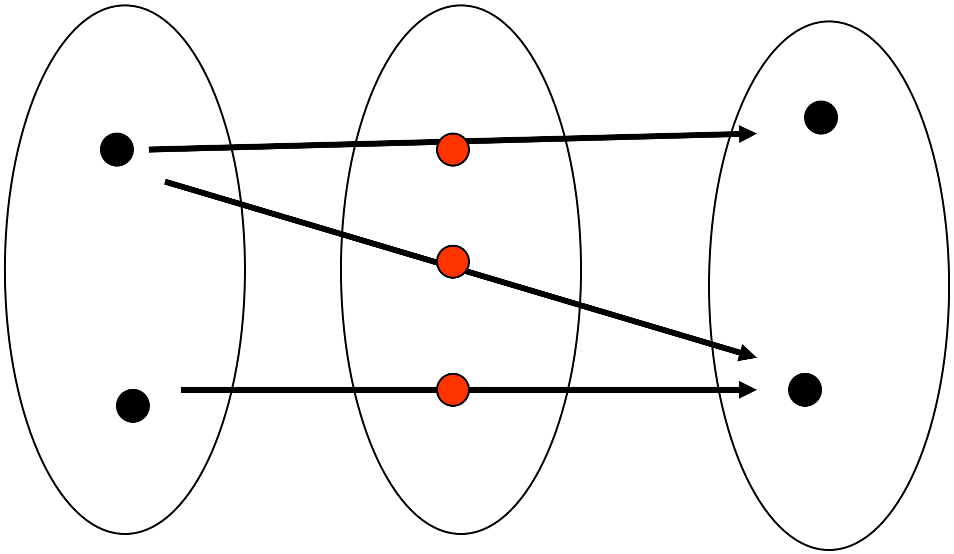
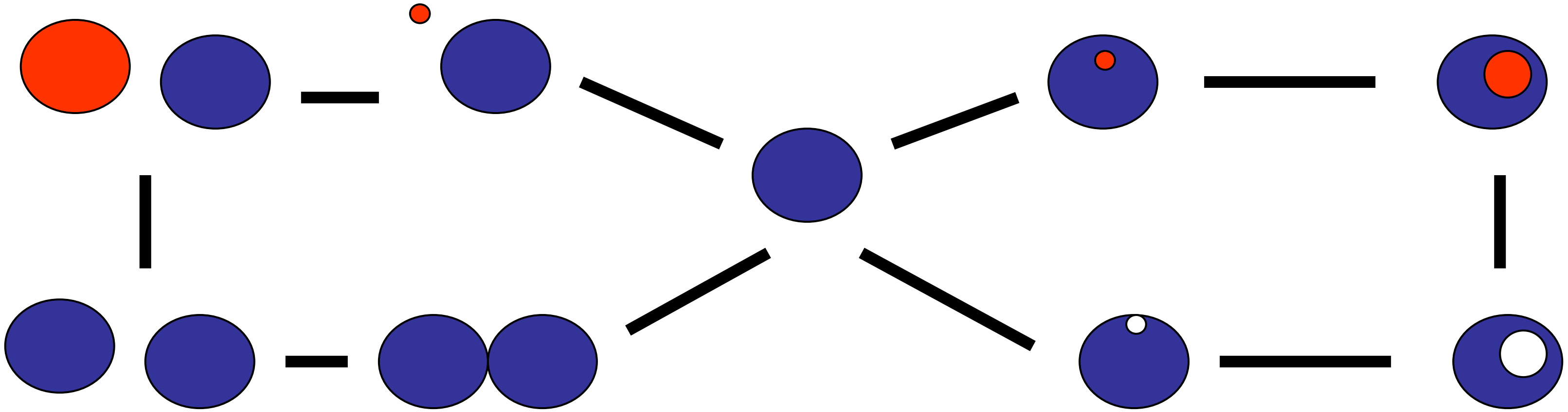


SPLIT MERGE

Another thought …

Conceptual neighborhood diagram



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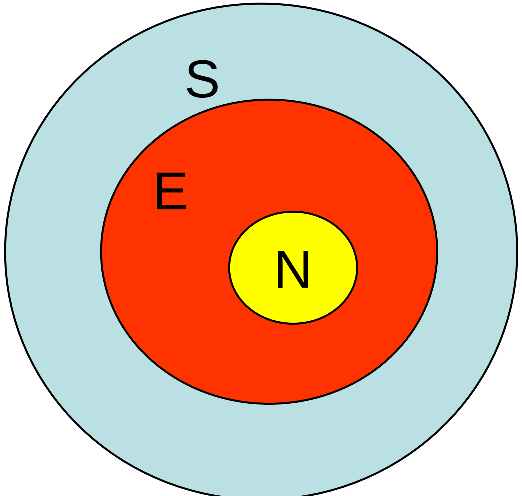
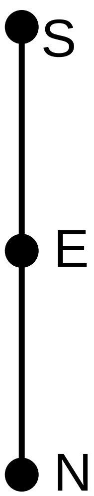
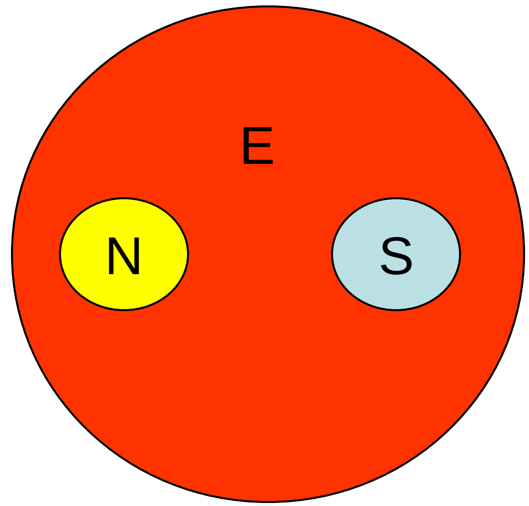
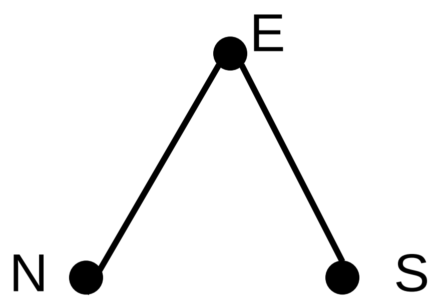
From plane to sphere









Furthermore:
merge = self merge (=fold)
split=self-split (=unfold)

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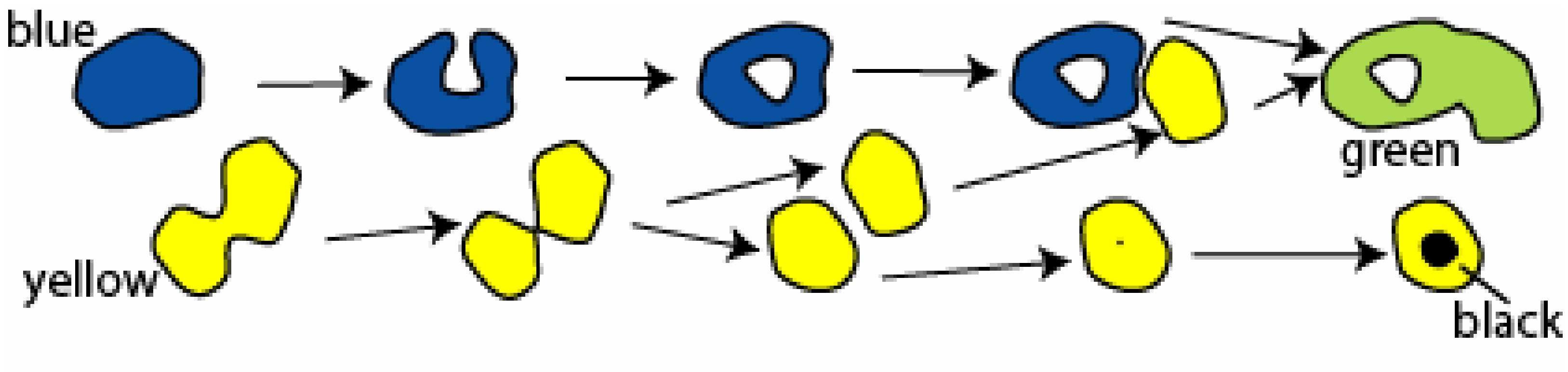
Even more thoughts

- Mixing:



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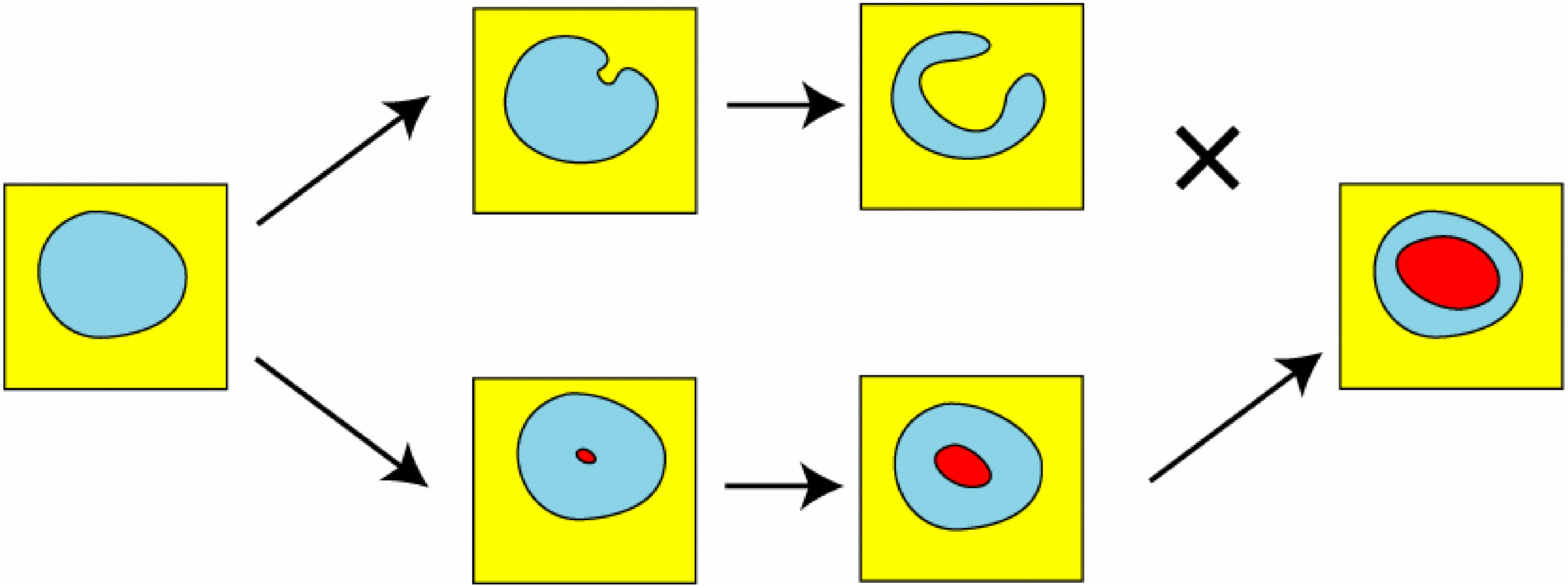
Different types of 'substance' involved in topological change



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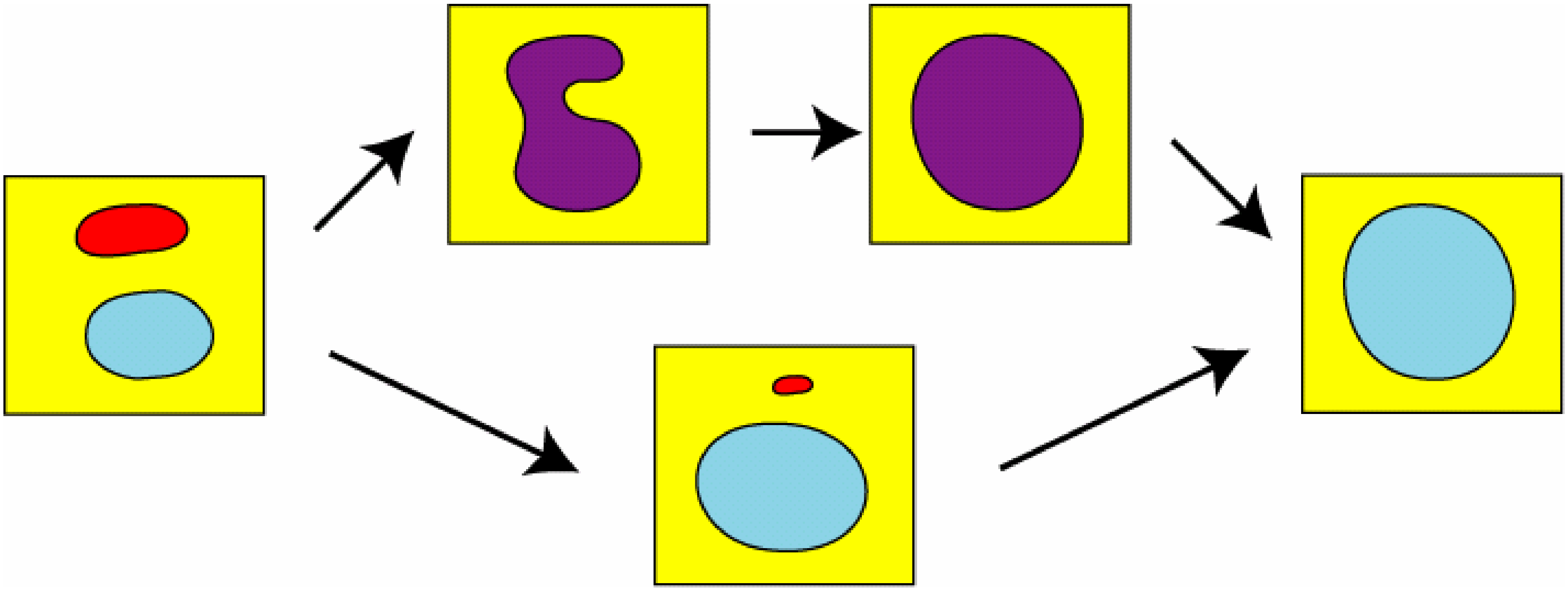
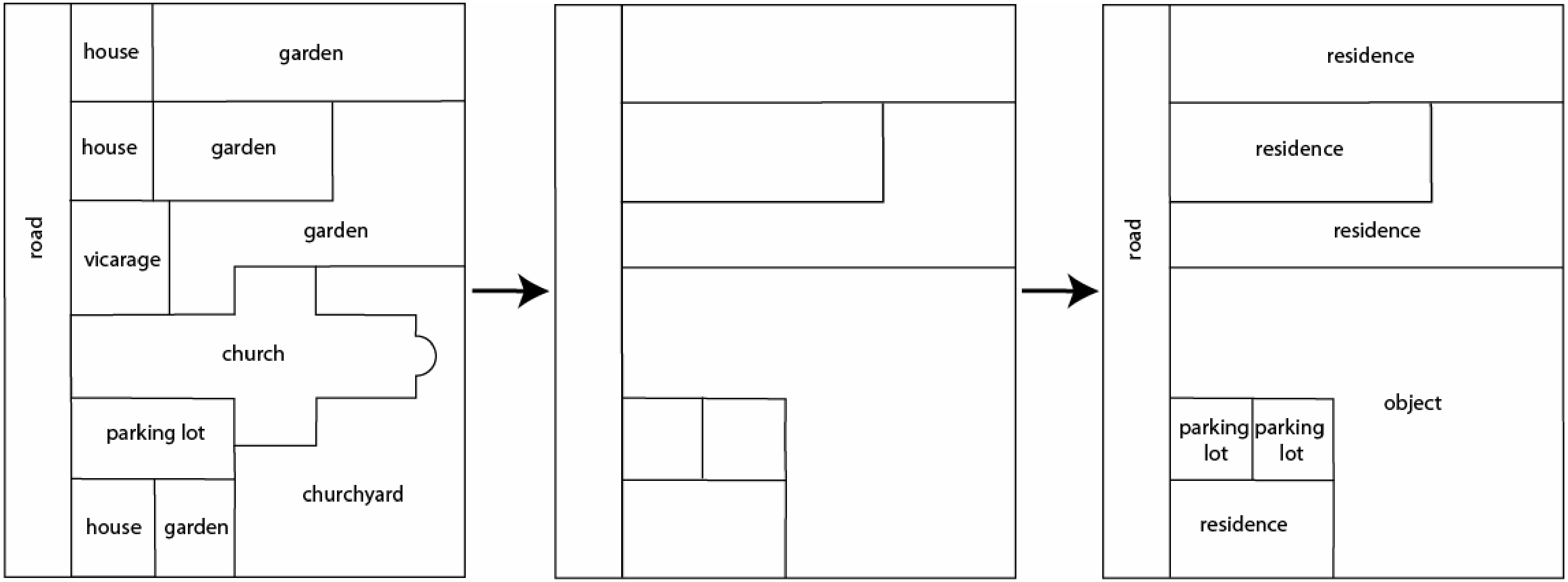
… or



Cartographic generalization



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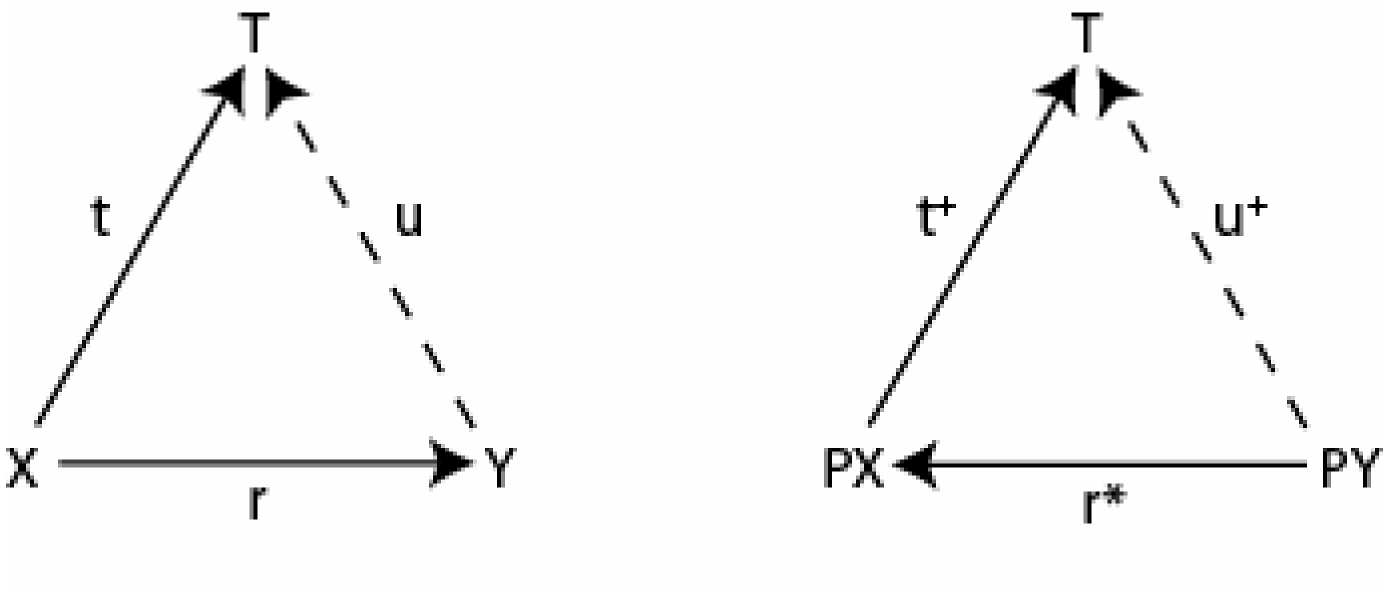
Formalization as labeled areas



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