Establishing Similarity Across Multi-Granular Topological-Relation Ontologies

Matthew P. Dube and Max J. Egenhofer

National Center for Geographic Information and Analysis
and
Department of Spatial Information Science and Engineering
University of Maine
Boardman Hall, Orono, ME 04469-5711, USA
matthew.dube@umit.maine.edu, max@spatial.maine.edu

Abstract. Within the Geospatial Semantic Web, selecting a different ontology for a spatial data set will enable that data's analysis in a different context. Analyses of multiple data sets, each based on a different ontology, require appropriate bridges across the ontologies. This paper focuses on establishing such a bridge across two topological-relation ontologies of different granularity—the standard *eight detailed* toplogical relations and *five coarse* topological relations. By mapping the conceptual neighborhood graphs onto a zonal representation, the different granularities are aligned spatially, yielding a reasoned approach to determining similarity values for the bridges across the two ontologies. A comparison with bridge lengths from an averaged model shows the better quality of zonal model.

1. Introduction

Geospatial ontologies that capture semantics of spatial information are paramount for the Geospatial Semantic Web (Egenhofer 2002, Fonseca and Sheth 2002) in order to enable consistent spatial retrieval and Web-based geospatial services. The choice of an ontology provides an opportunity to analyze data in a different context or from a different perspective (Fonseca and Egenhofer 1999). One of the major challenges that the Geospatial Semantic Web faces is when multiple spatial models need to be integrated, as in the case when data providers resort to using their own ontologies (i.e., data models and inference mechanisms) without providing explicit instruments to covert to other representations and reasoning methods. This paper deals with bridging across diverse spatial ontologies.

Some of the most advanced formalizations and agreements of spatial concepts are currently found in the domain of topological relations with sound formalisms (Egenhofer and Franzosa 1991; Randell *et al*. 1992). Since theses formalisms address abstract spatial models and inferences that permeate across all kinds of application domains they are typically considered essential ingredients for upper-level ontologies, such as OpenCyc (Lenat and Guha 1990) and the Standard Ontology for Ubiquitous and Pervasive Applications SOUPA (Chen *et al*. 2005).

While ontology alignment has been a stronghold in geospatial feature class consolidation (Cruz and Sunna 2008), less effort has been put into the development of methods for bridging between definitions of spatial relations, which differs from typical ontology alignments as the usual methods of lexical or structural comparisons (Euzenat and Shvaiko 2007) do not apply. Instead, a model-based integration is needed that exploits the inherent semantics of the underlying phenomena. This paper addresses the quality improvements of such alignments over a lexical or an *ad hoc* approach. Better knowledge about accommodating such spatial-relation ontologies across different granularities generalizes beyond the particular sets of relations studied and will lead to improved interoperability.

The purpose of this paper is to bridge spatial relation ontologies that address the same spatial concepts, but are defined at different granularities. The bridging should enable computations across the system divide. Such a system would allow for comparisons between coarse and detailed relations. Competing comparisons of coarse and detailed spatial relations may occur whenever two or more parties base their spatial analysis on different spatial relation ontologies. For example, from the perspective of a bank lending money to a potential new homeowner, the critical topological issue is whether the entire footprint of the building is contained on the land parcel, reflecting a coarse view of the topological relation *in* (disregarding whether parts of all of the building's boundary coincides with the land parcel's boundary). In a dispute with a neighbor about cleaning the house's gutters, however, a more detailed notion of inclusion applies as a boundary coincidence may require an easement, granting a right to step on the neighbor's property to maintain parts of the building. Comparisons across the two notions of inclusion require now conflict resolutions and mediations in the same way as the use of different spatial ontologies does for spatial entity classes (Fonseca *et al*. 2002).

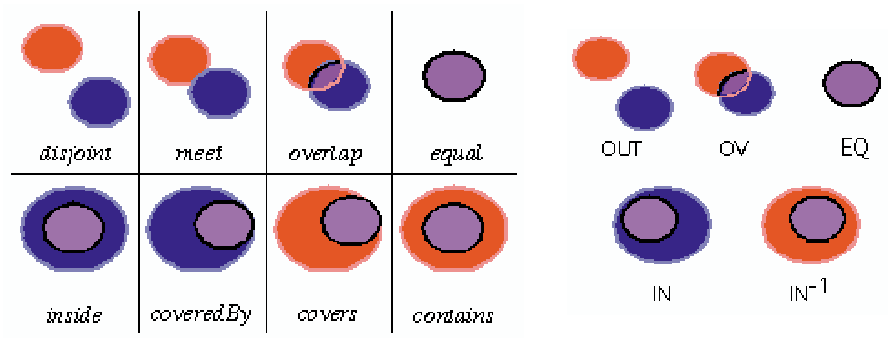
While logical inferences over the detailed and coarse relations, implied by the relations' composition table (Egenhofer 1991), have been integrated into description logic languages (Haarslev *et al*. 1999) so that they are executable by automated reasoners (Haarslev and Möller 2001), similarity reasoning over topological relations has been limited to relations of the same granularity (Egenhofer 1997). Such similarity inferences need not only address the linkages among the most similar relations, but also require quantifications of such similarities in order to determine the cost for constraint relaxations. For purely topological reasoning this difference between each pair of most similar relations has been the unit within a complete set of the same granularity.

The remainder of the paper is structured as follows: Section 2 briefly reviews the sets of detailed and coarse topological relations and their conceptual neighborhood graphs. Section 3 analyzes how these graphs should be aligned. Section 4 introduces the zonal representation of the relations' neighborhoods and derives the lengths of the bridges across the relation ontologies. Section 5 analyzes the quality of the so established bridges and their lengths. The paper closes in Section 6 with conclusions and a discussion of future work.

2. Models for Topological Relations

The 9-intersection (Egenhofer and Herring 1990) and RCC, the Region-Connection Calculus (Randell *et al*. 1992), establish an ontology for topological spatial relations. The two methods yield equivalent results when the relations' range is restricted to 2-dimensional objects that are homeomorphic to regularized closed 2-disks (i.e., each region's closure is identical to the closure of the region's interior). For such simple regions, RCC and the 9-intersection identify a set of eight jointly exhaustive and pairwise disjoint relations, subsequently referred to by their 9-intersection terminology (Figure 1a).

Since the eight base relations form a relation algebra (Smith and Park 1992), disjunctions of these relations lead to *coarse* topological relations. A common generalization of the eight region relations is the RCC-5 abstraction (Bennett 1994), in which the relations *inside* and *coveredBy* are generalized to a single relation, here called IN, *contains* and *covers* generalize to IN^{-1}, *disjoint* and *meet* are combined into OUT, while the remaining two relations *overlap* and *equal* are mapped 1:1 onto OV and EQ, respectively (Figure 1b). These coarse relations have then the following interpretations: OUT means that both regions' interiors do not intersect; for OV regions share a portion of their interiors and exteriors with the opposing interior and exterior; for EQ the regions have coincident interiors; IN means that one region is a proper subset of the other; and IN^{-1} is converse to IN. Other mappings onto coarser topological relations—not considered here—consist of another domain over five relations with somewhat different mappings (Grigni *et al*. 1995) or a domain with two relations (essentially distinguishing *equal* and its complement not *equal*).



(a) (b)

Fig. 1. Topological relations: (a) the eight detailed relations, (b) the five coarse relations.

While the topological relations are on a nominal scale of measurements, the least amount of change needed to transform one configuration into another yields an order that is captured by the relations' conceptual neighborhood graph (Egenhofer and Al-Taha 1992, Cui *et al*. 1992). Similar to Allen's (1983) interval relations where different rationales for establishing neighborhoods lead to different neighborhood graphs (Freksa 1992), variations of the region relations' conceptual neighborhood graph can be found (Cohn *et al*. 1994). In the remainder of this paper only relation pairs captured by the A-neighborhood (based on translation, rotation and scaling free of any metric constraints on the involved regions, such as the regions' shapes) are considered (Figure 2a). In a similar way the eight coarse relations can be aligned forming their conceptual neighborhood graph (Figure 2b).

(a) (b)

Fig. 2. The conceptual neighborhood graphs of (a) the eight detailed relations and (b) the five coarse topological relations.

3. Aligning Detailed and Coarse Topological Relations

The conceptual neighborhoods form the foundation for qualitative similarity reasoning (Bruns and Egenhofer 1996) as a relation's most similar relations correspond to that relation's neighbors in the graph, and increasingly less similar relations are neighbors of higher degrees. When reasoning about detailed *and* coarse topological relations, it is necessary to connect the two graphs. Such ontology alignment can be accomplished with a subset of the five binary semantic relations between two concepts (Euzenat 2008). It has two 1:1 mappings and six inclusions (Figure 3). These mappings build bridges in the graph between the detailed and the coarse relations (Figure 4). Computational similarity models require not only the association of most similar pairs, but also a quantification of the similarity (Rodríguez and Egenhofer 2003) in order to support consistent reasoning.

OV ≡ *overlap* ⇒ OV = *overlap*	EQ ≡ *equal* ⇒ EQ = *equal*
OUT ≡ *disjoint* ∨ *meet* ⇒ OUT > *disjoint*	OUT ≡ *disjoint* ∨ *meet* ⇒ OUT > *meet*
IN ≡ *inside* ∨ *coveredBy* ⇒ IN > *inside*	IN ≡ *inside* ∨ *coveredBy* ⇒ IN > *coveredBy*
IN^{-1} ≡ *contains* ∨ *covers* ⇒ IN^{-1} > *contains*	IN^{-1} ≡ *contains* ∨ *covers* ⇒ IN^{-1} > *covers*

Fig. 3: Mapping detailed onto coarse topological relations using the semantic relations inclusion (<) and equivalence (=).

The mere alignment, however, offers no support for analytical operations such as similarity assessments involving detailed *and* coarse topological relations, which rely on the distances between the relations within the embedding of the neighborhood graph. For the isolated conceptual neighborhood graphs the distances along the graphs' edges are typically assumed to be of length 1 if only topological changes are considered. With this metric, path lengths can be established, which lay the foundation for determining quantitatively differences between pairs of qualitative topological relations (Egenhofer 1997). Likewise these distances yield shortest paths, which characterize the least amount of differences between two topological relations. These same metric assignments to the bridges across the two ontologies would, however, lead to undesired side effects. First, the distance for pairs of equivalent relations (i.e., *overlap* and OV as well as *equal* and EQ) must be 0. Second, for bridges from two detailed to a coarse relation—that is, the disjunction of two detailed relations (Figure 3)—a distance of 1 would mean that for one half of the detailed relations the distance to the coarse relation matches with the detailed relations, while for the other half it would yield a contraction. A third choice assigns to each bridge classified by an inclusion relation (<) a distance of 0.5, and 0 to each bridge with an equivalence relation (=). This model, called the *50% model*, is used in Section 5 as a basis for assessing the quality of the model developed in Section 4.

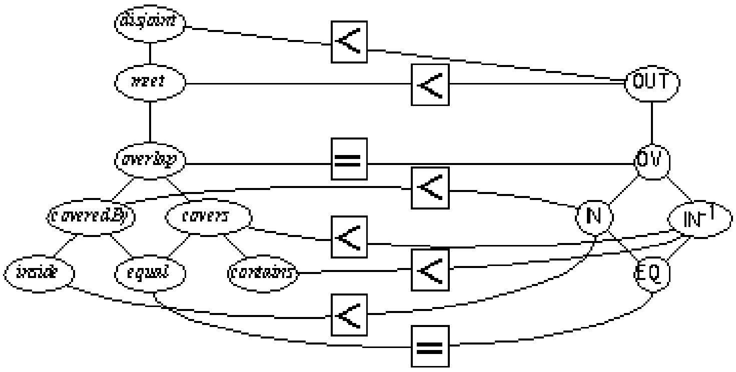


Fig. 4. The aligned conceptual neighborhood graph of the eight detailed relations and the five coarse topological relations with the semantic relations inclusion (<) and equivalence (=).

4. The Zonal Representation for the Neighborhood of Detailed and Coarse Topological Relations

We introduce for topological relations their *zonal representation*, which uses circles to capture a relation's range of influence and represents the similarities between region relations in the conceptual neighborhood graph (Figure 5a). Coarse relations that are not used as bridges need to fit somewhere into these circles. These circles are referred to by the name of the first relation from *overlap* contacted in the detailed topological relations. For example, any relation without an interior-interior intersection falls into the *meet* circle. Since *equal* is an intersection of *coveredBy* and *covers*, it falls on the intersecting point of these two circles. The next step connects *overlap* to the nodes for *meet*, *covers*, and *coveredBy*, and then to *disjoint*, *contains*, and *inside* (Figure 5b).

Each circle around *meet*, *coveredBy*, and *covers* has a radius of 1, therefore, all detailed relations maintain their initial similarity values from the conceptual neighborhood graph. This graph is actually just imposed over the circles. The purpose of the circles is to encapsulate the combinations among the coarse topological relations and to derive a distance measure for them. The zones that represent the coarse relations' combinations are shown in Figure 4c. Subsequently the *covers*-zone is used to exhibit the procedures for deriving the distances, since the remaining two zones follow suit under rotation.

Unlike the detailed topological relations *equal*, *overlap*, *contains*, and *covers*, the coarse relation IN^{-1} is represented as a zone rather than a node. It is desirable to come up with a node to represent the coarse relation so that a similarity measure can be assigned between it and any other relation in the mapping between coarse and detailed topological relations.

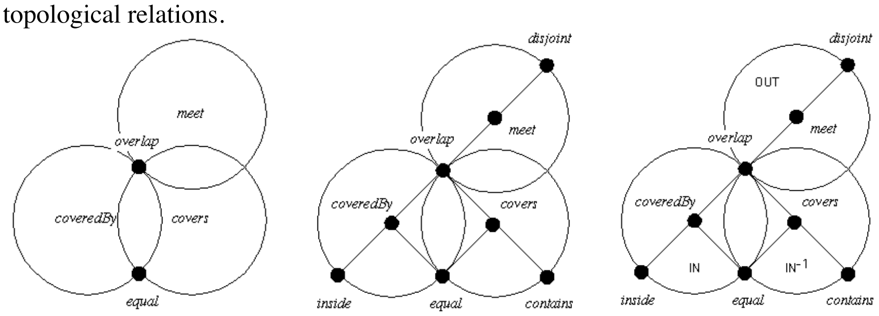


Fig. 5. (a) Zonal configuration for the detailed and coarse topological relations, (b) the conceptual neighborhood graph fused with the zones, and (c) the coarse relation constructs denoted on the zones.

The first distance to isolate is the distance between *covers* and IN^{-1}. This is rather straight forward as *covers* lies at the center of the unit circle. The double integral of the distance function over the unit circle from its center is $\pi/6$. Also the area of the first quadrant of the unit circle, representing the sector <*equal*, *covers*, *contains*> is $\pi/4$. The average distance from *covers* to any point in the zone denoted as IN^{-1} is then $(\pi/6)/(\pi/4)$, that is, 2/3 (Figure 6a).

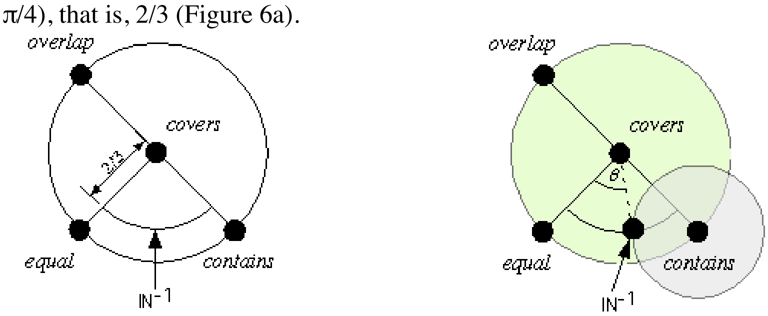


Fig. 6. (a) Arc representing IN^{-1} and (b) node for IN^{-1}.

The next distance necessary is the distance from *contains* to IN^{-1}. The key to resolving this issue is how to place IN^{-1} with respect to its exemplars *contains*, *covers*, and *equal*. From a cognitive perspective, *contains* should be closer to IN^{-1} than *covers* or *equal*, because it is more of the prototype of the containment relation than *covers* or *equal*. Let A be the portion of the perimeter of the *covers* circle with radius 2/3 in the first quadrant, such that x (i.e., a point on the perimeter) is within 2/3 of the point

(0,1), which represents *contains*. The desired value is the average distance from *contains* to any point in A. A is bounded by the x-coordinates 0 and $(2/3)\cdot\cos(\theta)$=0.441, with $\theta \approx 0.848$ so that that the point on the perimeter is 2/3 away from (0,1). The distance between contains and IN^{-1} is computed by the mean value theorem, that is, calculating the integral of the distance formula between point (0,1) and the circle of radius 2/3 and then dividing by the bounds 0 and 0.441. It yields approximately 0.454.

The measures from *covers* to IN^{-1} and from *contains* to IN^{-1}, and the constraint of the first quadrant, tie down the position of IN^{-1}—as the intersection of the circle with radius 2/3 around *covers* with the circle of radius 0.454 around *contains*—and only leave the third distance (from *equal* to IN^{-1}) to be calculated (Figure 6b).

Intersecting the circles, the location of the node for IN^{-1} in relation to the zone is (0.247, 0.619). The distance between this point and the node for *equal* (1,0) is 0.975. Figure 7a shows the configuration with all distances among the relations *overlap*, *covers*, *equal*, *contains*, and IN^{-1}. All distances can be translated to the other zones (*meet* and *coveredBy*), yielding the aligned and quantified neighborhood graph for the combined detailed and coarse relations (Figure 7b). The three distances calculated under this method satisfy the triangle inequality; therefore, the aligned graph is a metric space.

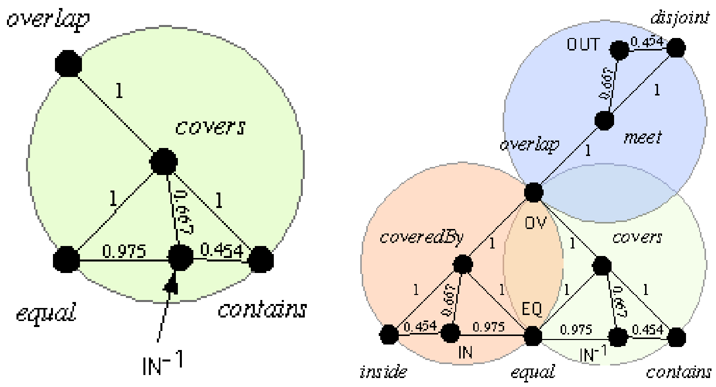


Fig. 7. Similarities between (a) IN^{-1} and *covers*, *equal*, *contains*, overlap, and (b) between the thirteen detailed and coarse relations.

5. Quality of the Aligned Neighborhood Graphs

Several psychological findings guide the quality of such alignments. According to the *prototype theory*, people typically assign a dominant member of the superclass, becoming a salient image, and then compare all other objects against this prototype. Alternatively, the most central member of the superclass is selected as the prototype (Rosch 1973). A competing theory is the *exemplar theory* of psychology, which starts with a basis of observations, called *exemplars*, that are known whether or not they belong to a set. Any subsequent objects then become compared to this group of objects to discern membership in a superclass (Medin and Schaffer, 1978). Another related theory is Wittgenstein's (1953) *graded sets*, which assign a hierarchy of dominance to members of a superclass, because not all members of the set are considered to give an adequate representation of the whole. In order to adequately perform such alignments, both the prototype theory and the exemplar theory must be taken into account by establishing a bounding set representing the coarse relations (exemplar theory), establishing a hierarchy of closest neighbors from the detailed up to the coarse relations (graded set), and from this graded set, discerning the prototypical member (prototype theory). Bounding relations from the detailed topological relations already have been established, providing a methodical basis by which to formulate scene similarities between other systems with inherent bridges or other systems, which are essentially disjunctions of members of the detailed relations.

The complete set of distances for the Cartesian product of detailed and coarse relations (Figure 8) reveals a number of properties of the aligned similarity graphs:

- The shortest distance between any two different relations is 0.454, which has three occurrences (always between a detailed and a coarse relation and, therefore, part of the alignment). This value is less than the standard distance between two coarse or two detailed relations in the isolated neighborhood graphs.
- The longest distance between two relations is 4, the same as the longest distance in the isolated neighborhood graph of detailed relations; therefore, the alignment has no effect on the longest shortest path among the detailed relations.
- The distances of the bridges from detailed to coarse relations range from 0 (since *equal* = EQ and *overlap* = OV) to 0.667. The upper bound is considerably less than the standard distances between neighboring pairs in the isolated graph of detailed relations and the isolated graph of coarse relations. Apparently a homogeneous choice of 1 as the length of all edges of the aligned graphs would be counter to the geometric arguments that emerged out of the zonal model.
- The post-alignment distances among the eight detailed relations coincide with their corresponding pre-alignment distances in 54 out of 64 cases (which corresponds to an 82% agreement when accounting for implied values of converse cases and the distance between each relation and itself). For the five detailed relation pairs that are impacted by the alignment (*equal–inside*, *contains–covers, contains–inside*, *equal–contains*, *inside–coveredBy*) the post-alignment distance is shorter than the pre-alignment distance.
- The post-alignment distances among the five coarse relations coincide with their corresponding pre-alignment distances in 7 out of 25 cases, that is, only one distance—between OV and EQ—remains unchanged when accounting for implied values of converse cases and the distance between each relation and itself. Three post-alignment distances (and their reverse distances) are shorter than their corresponding pre-alignment distances—IN–IN^{-1}, IN–EQ, and IN^{-1}–EQ—whereas the remaining six relation pairs—OUT-OV, OUT-IN, OUT-IN^{-1}, OUT-EQ, OV-IN, and OV-IN^{-1}—show increases in their post-alignment distances.

	disjoint	*meet*	*overlap*	*coveredBy*	*covers*	*inside*	*contains*	*equal*	OUT	OV	IN	IN^{-1}	EQ
disjoint	0	1	2	3	3	4	4	4	A	2	3+B	3+B	4
meet	1	0	1	2	2	3	3	3	B	1	2+B	2+B	3
overlap	2	1	0	1	1	2	2	2	1+B	0	1+B	1+B	2
coveredBy	3	2	1	0	2	1	1+ A+C	1	2+B	1	B	1+C	1
covers	3	2	1	2	0	1+ A+C	1	1	2+B	1	1+C	B	1
inside	4	3	2	1	1+ A+C	0	2A+ 2C	A+C	3+B	2	A	A+2C	A+C
contains	4	3	2	1+ A+C	1	2A+ 2C	0	A+C	3+B	2	A+2C	A	A+C
equal	4	3	2	1	1	A+C	A+C	0	3+B	2	C	C	0
OUT	A	B	1+B	2+B	2+B	3+B	3+B	3+B	0	1+B	2+2B	2+2B	3+B
OV	2	1	0	1	1	2	2	2	1+B	0	1+B	1+B	2
IN	3+B	2+B	1+B	B	1+C	A	A+ 2C	C	2+2B	1+B	0	2C	C
IN^{-1}	3+B	2+B	1+B	1+C	B	A+2C	A	C	2+2B	1+B	2C	0	C
EQ	4	3	2	1	1	A+C	A+C	0	3+B	2	C	C	0

Fig. 8. Dissimilarities between all pairs of coarse and detailed topological relations (A=0.454; B=0.667; C= 0.975).

As a quantitative base for the assessment of the aligned neighborhood graph we investigate the relations along the neat line of the two isolated graphs and their bridges. Two bridges create two paths from the detailed to the coarse relation—one by first crossing the bridge, followed by a transition to the coarse target relations and the other by first making the transition to the linked detailed relation, followed by crossing the bridge from there to the coarse relation. More similar path lengths are considered as an indicator of better alignments. We selected three such pairs of bridges and compared the path lengths for the zonal model (Section 4) and the 50%-model (Section 3). In two cases the zonal model yielded smaller differences than the 50%-model, and in one case both featured the same path length differences, which supports the zonal model's quality alignment (Figure 9).

Path	Zonal Model		50%-Model	
	path length	path length diff.	path length	path length diff.
disjoint-OUT-OV	2.121	0.121	1.5	0.5
disjoint-overlap-OV	2		2	
overlap-OV-IN	1.667	0.787	1	1.5
overlap-inside-IN	2.454		2.5	
overlap-OV-EQ	2	0	2	0
overlap-equal-EQ	2		2	

Fig. 9. Path length differences in the zonal and 50%-model

6. Conclusions

We developed a method to attach two topological-relation ontologies of different granularities so that the bridges across the ontologies carry lengths that are compatible with the relations' conceptual neighborhood graphs. Such alignments of multi-granular ontologies will contribute to analyzing spatial data in different contexts. The comparison with other distance choices showed the pointed advantage of the zonal model. A number of open questions remain for future work:

- How does the method perform if each neighborhood graph is first normalized by longest shortest path prior to bridging?
- How does the coarse relation alignment differ from attaching a somewhat differently defined set of five coarse topological relations (Grigni *et al*. 1995)?
- How would an even coarser set of only two relations (*same* and *different*) behave in this process, particularly with respect to transitivity?

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