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Multi-Resolution Spatial Databases: Consistency Among Networks[*]

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Abstract

Heterogeneous geographic databases contain multiple views of the same geographic objects at different levels of spatial resolution which must be consistent with each other. This paper develops a formal method of assessing consistency of topological relations in R^2 at a high level of abstraction, over complexly structured spatial objects that form networks. Networks are a frequently used data type in geographic applications where, for instance, a river with its tributaries and lakes is considered a single spatial object. The relationships between such a network and another spatial object are often used as criteria for describing the consistency in multi-scale geographic applications. The proposed model includes computational methods to determine when two networks are the same or not. The model is further extended to cover the relationships between a network and another spatial object, such as a region or a line. These methods will play an important role in the assessment of spatial similarity of scenes with complex configurations including roads and urban areas, paths, and other linear geographic features.

1. Introduction

Generalization is an indispensable part of cartographic production, spatial decision support systems, and other scientific branches constituting the context of Geographic Information Systems (GISs). It results in multiple representations, occasionally thought of as *copies*, of the same scenes of objects at different levels of spatial resolution, also referred to as *scale* [MLW95]. The same objects are represented in several different ways, tailored towards the needs of different users and analyses.

GIS users perform such operations as zooming into and out of a map-like representation, and expect that the predominant spatial relations be preserved throughout such scale changes. Likewise, a query evaluation algorithm executed at a coarser level should give the same, or at least a very similar, result as a query processed against a more detailed level [BEF92]. Cartographic map series at different scales are another example of applications that need multiple representation levels. The key to the successful adoption of multi-resolution geographic databases is the maintenance of *consistency* across the different representations. *Consistency* describes the absence of any logical contradictions within a model of reality. It is an important criterion for geographic databases, especially when map production is involved. For example, the U.S. National Ocean Service (NOS) maintains a two-tier map production system, ANCS-II, which allows cartographers to derive and maintain chart panels at different scales for the same geographic area from a single geographic database [N94]. This cartographic database requires map generalization to produce the location and appearance of features to be placed at charts of different scale. NOS's guidelines

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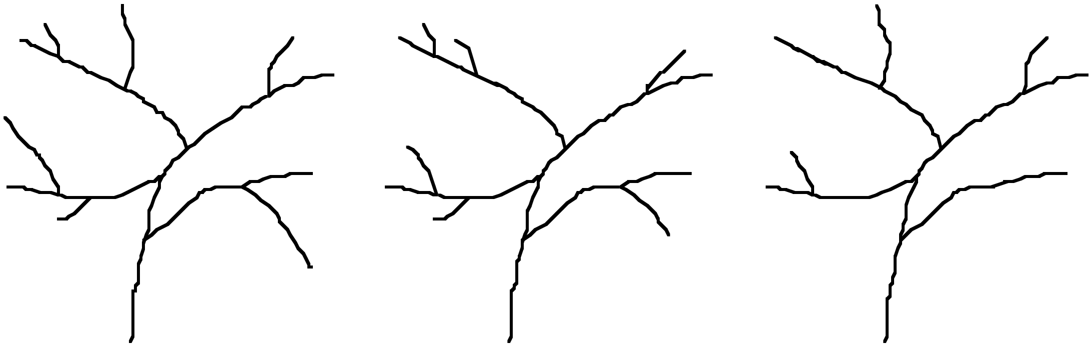
for this generalization process demand that a feature present at a small scale also be represented at any larger scale of the same area. Although there has been considerable research in recent years on cartographic generalization [BM91, MLW95] and various aspects of multiple representations [BK91, PD95, RS95, TVP+92], we currently lack methods to consistently maintain multi-resolution geographic databases. Other efforts in representing consistently geographic data only deal with one level of detail [P96, US96].

Currently, *multi-resolution geographic databases* record explicitly multiple representations covering the same geographic area at different scales. Such databases require a mechanism to detect inconsistencies among the different representations. In the future multi-resolution geographic databases are envisioned that would derive multiple representations “on the fly,” providing more flexibility such as continuous zooming. Under such a scenario, quality control mechanisms will be needed to confirm that a generalization algorithms produces consistent results. The methods discussed in this paper apply to both scenarios.

Assessment and maintenance of consistency among multiple copies of geographic data requires geographic domain knowledge. This paper focuses on domain knowledge about networks, an important spatial data type shared by many geographic applications such as hydrology [MG82] and transportation [TVP+92]. Consistency of scenes representing networks at different levels of detail, has to be considered under two main perspectives: (1) when small changes occur from the one level to the next, and (2) when large changes take place. Figure 1b shows a network with small changes compared to the network in Figure 1a, because only the lengths of segments and the angles between segments have been modestly modified. On the other hand, the network in Figure 1c shows a more dramatic change, because segments are dropped or aggregated. This paper addresses the checking of consistency for small-scale changes. Such consistency assessment s mean primarily to test for topological equivalence of the objects and their relations [ECF94].



(a) (b) (c)

Figure 1: Small scale changes between (a) to (b) small changes (length of segments, angle between segments), and large changes between (a) to (c) (aggregation or elimination of segments).

Current models for spatial relations among objects only apply to simple, homogeneously n-dimensional objects, i.e., each object must be either a region [EF91] or a line [E93a], but not a compound of them. This paper develops a comprehensive theory for dealing with spatial relations in R^2 involving heterogeneous, connected objects, at a high level of abstraction, i.e., when small changes occur. Such topological relations focus on the most salient properties of a spatial configuration, and ignore such metric properties as the length or shape of a line segment, the angle between two line segments, or the relative size of an areal feature. Several other metric or pseudo-metric measures may play a role in the determination of spatial configurations. They are complementary to the model introduced here and may provide more details as a refinement of topology. The method employed is based on the *9-intersection model* [EH91] and the *boundary-boundary components sequence* [EF95]. This formalization is part of a larger research effort developing formal models for checking the consistency among multiple representations in heterogeneous geographic databases.

The remainder of the paper is organized as follows: Section 2 summarizes the background for topological relations in R^2, defines homogeneous and heterogeneous networks, and provides a means to describe a scene of networks. Section 3 describes the consistency of scenes of homogeneous networks and

Section 4 the consistency of scenes of heterogeneous networks. Section 5 builds on these results and shows how consistency among scenes, each one containing homogeneous networks and regions, can be assessed. Section 6 extends these results to detect consistency of scenes of heterogeneous networks and regions. Section 7 focuses on scenes of heterogeneous networks and lines. Section 8 concludes with a summary of the results and a discussion of future work.

2. Topological Relations

The foundation of the assessment of topological consistency of relations involving networks is based on the topological relations of the networks' constituent parts, i.e., lines and regions and a formal model for heterogeneous connected objects, introducing component invariants, such as *intersection orientation* and *boundary sequence*. This section describes this background and provides a means to describe a scene with heterogeneous objects.

2.1 The 9-Intersection Model

The model used for topological relations is the 9-intersection [EH91]. The basic components of a spatial object A are its *interior* A°, *boundary* ∂A, and *exterior* A^-. The 9-intersection model describes the topological relations between objects based on their *interior*, *boundary*, and *exterior.* Within the context of networks, we will need the topological relations between two regions, between a region and a line, and between two lines. The boundary, interior and exterior of a simply connected line are defined in accordance with algebraic topology [S66]. The boundary comprises the line's endpoints, the interior is the line's closure minus the boundary, and the exterior is the complement of the line's closure.

2.2 A Model for Networks

This section introduces the notion and concepts of *networks*. We distinguish between *homogeneous* networks, which are made up of line segments only, and *heterogeneous* networks, which contain linear and areal objects. A canonical representation of networks aids in the assessment of their topological structure, and for this goal we introduce a set of normalization rules for the constituent objects—line segments and regions—that are necessary to form valid networks. Additionally, in order to describe more topological details several component invariants are discussed: *dimension* and *sequence* of *boundary-boundary components*.

2.2.1 Homogeneous Networks

Homogeneous networks are defined in terms of line segments.

Definition 1: A *line segment* A—or just *segment*—is a continuous, non-intersecting sequence of points (x,y) in R^2 such that the segment's boundary is disconnected.

The segment's boundary is sometimes also referred to as the line segment's *endpoints.* Segments may be straight lines or lines that may have shape points, but they must not self-intersect nor may a single line segment contain cycles (Figure 2).

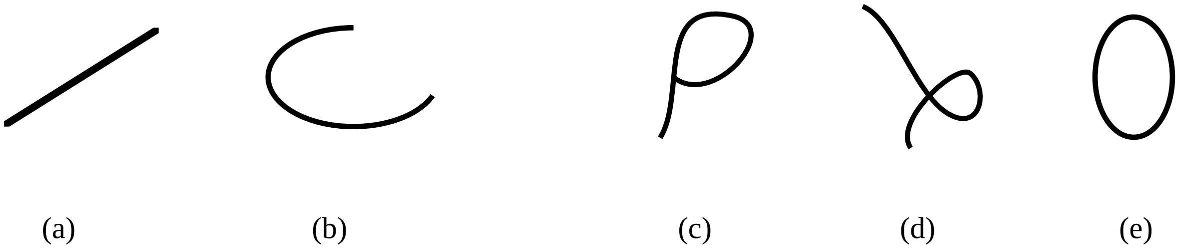


Figure 2: Sequences of points that are (a-b) valid line segments and (c-e) invalid line segments.

There are two basic topological relations between two distinct segments[1]: two segments are *disjoint* if neither boundaries nor interiors intersect (Figure 3a) and they *meet* if the segments only intersect in their boundaries, but not their interiors. For *meet* we distinguish two types: the relation between segments that *meet* in one endpoint will be called *#1-meet* (Figure 3b), while segments that *meet* in both endpoints will be called *#2-meet* (Figure 3c).

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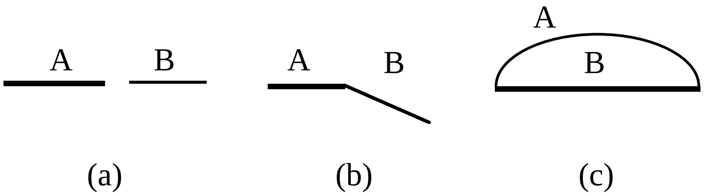
[1] In addition, each segment is *equal* to itself.



(a) (b) (c)

Figure 3: Topological relations between two segments A and B: (a) *disjoint*, (b) *#1-meet*, and (c) *#2-meet*.

A critical property for segments in a network is their *connectedness*.

Definition 2: Two segments A and B are *connected* if they meet *directly* (Equation 1a) or *indirectly* (Equation 1b).

connected (*A*, *B*) if *meet* (*A*, *B*) (1a)
connected (*A*, *B*) if (*meet* (*A*, *C*), *connected* (*C*, *B*)) (1b)

The point at which a set of line segments *meet* will be called the *intersection*. Around an intersection, the directly connected line segments (Equation 1a) are cyclically ordered. Given a consistent orientation of the plane—here we choose a counter-clockwise orientation—one can determine for each intersection the *intersection orientation* of its directly-connected segments. For example, the intersection orientation of the five directly-connected segments in Figure 4 is <*A*, *C*, *E*, *B*, *D*>. Depending on the choice of the starting segment, different orientation sequences are obtained. For the intersection in Figure 4, it would have been equally valid to start the sequence at any other segment, giving four equivalent alternatives: <*C*, *E*, *B*, *D*, *A*>, <*E*, *B*, *D*, *A*, *C*>, <*B*, *D*, *A*, *C*, *E*>, and <*D*, *A*, *C*, *E*, *B* >. In order to compare two orientations for equivalence it is necessary to perform an exhaustive cyclic permutation of the orientation sequences.

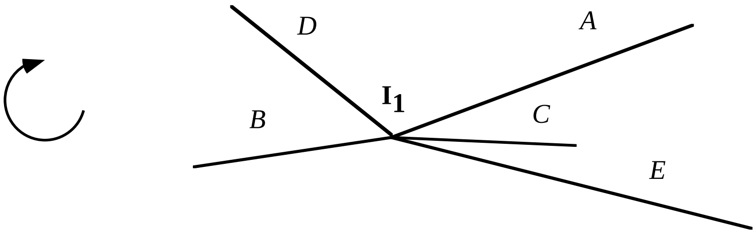
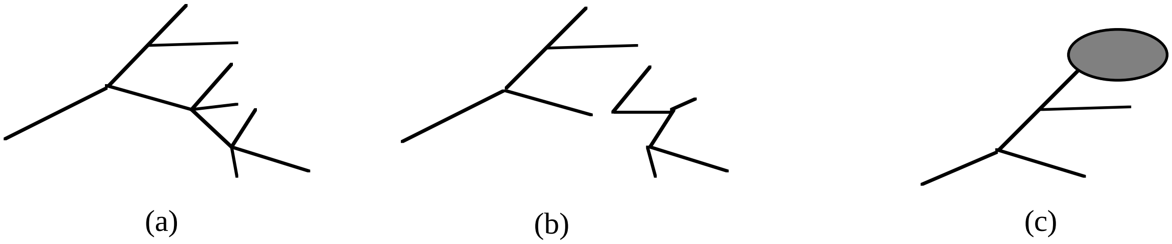


Figure 4: The orientation around intersection I_1: <*A*, *C*, *E*, *B*, *D*>.

Sets of line segments are *normalized* so that they are represented by a small set of segment relations—*disjoint, #1-meet,* and *#2-meet.*

Definition 3: A *homogeneous network* N is a set of connected segments S_i such that all segments are exclusively related by *#1-meet*, *#2-meet*, or *disjoint* relations and all intersections have at least three directly-connected segments.

Figure 5 shows examples of different configurations, some of which qualify as homogeneous networks, while others do not.



(a) (b) (c)

Figure 5: Configurations that (a) qualify as homogeneous networks and (b-c) do not qualify, because they are disconnected (b) or contain elements other than segments (c).

2.2.2 Heterogeneous Networks

Heterogeneous networks consist of line segments *and* regions. As such, they may be considered to be treated as extensions to homogeneous networks. Similar to the constraints about homogeneous networks, we request that a heterogeneous network must be connected and normalized. Heterogeneous networks have the same relations between segments as homogeneous networks; however, additional relation constraints exist between regions, and between a region and a line.

Regions in a heterogeneous network are either *disjoint* or they *meet*[2] (Figure 6). Depending on the number (n) of boundary-boundary components, the regions may be classified as *#n-meet* [E93b].


(a) (b)

Figure 6: The two topological relations between two regions in a heterogeneous network: (a) *disjoint* and (b) *meet*.

Likewise, any region in a heterogeneous network is either *disjoint* from a line segment, or it *meets* the segment such that either one or both boundaries of the segment coincide with the region's boundary, called *#1-meet* and *#2-meet*, respectively (Figure 7).

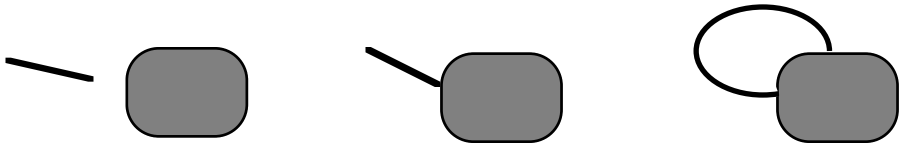

(a) (b) (c)

Figure 7: The three types of line-region relations (a) *disjoint*, (b) *#1-meet*, (c) *#2-meet*.

Segments and regions are cyclically ordered along the boundary of a region [EF95]. This *boundary sequence* corresponds to the intersection orientation for an intersection point of several line segments (Section 2.1). Given a clockwise orientation of the plane, the sequence is determined by starting at one boundary intersection, recording the region or segment that meets, and continuing with this process in the direction or the orientation until the original boundary intersection is reached again. If several segments meet in the same intersection, they are grouped together, preserving the order of the orientation. Figure 8 shows a configuration whose clockwise boundary sequence is $<S_1, S_2, S_1, <S_3, S_4> R_2, S_5, R_2, S_6, S_6>$.

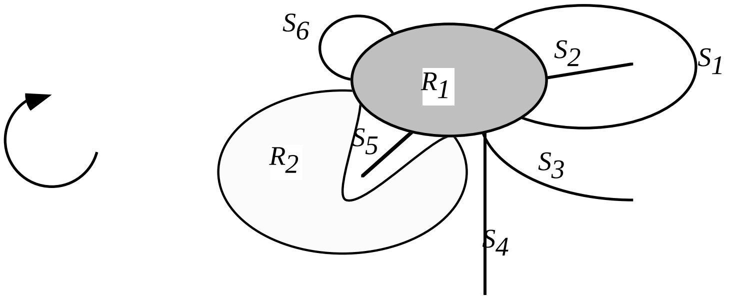


Figure 8: The boundary sequence around region R_1: $<S_1, S_2, S_1, <S_3, S_4>, R_2, S_5, R_2, S_6, S_6>$.

Since the choice of the starting segment or a starting region matters, two boundary sequences are equivalent if they match under cyclic permutations [EF95].

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[2] In addition, each region is *equal* to itself.

Definition 4: A *heterogeneous network* is a set of connected segments and regions such that all distinct segments are related by *#1-meet*, *#2-meet*, or *disjoint* relations; all distinct regions are *disjoint* or *meet*; and all segments are *disjoint* or *meet* all regions.

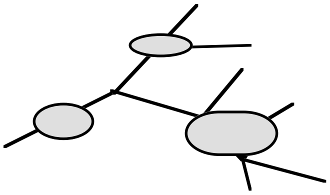

Figure 9: Example of a heterogeneous network.

3. Consistency of Scenes with Homogeneous Networks

In this section we present a method to assess topological consistency among scenes with homogeneous networks, when small changes occur, i.e., topological equivalence of networks has to be detected.

The topology of a network N with $S_1,\ldots,S_n$ segments and regions is fully specified by n^2 topological relations. These relations will be represented in an $n \times n$ matrix M, called the *relation matrix*.

Two homogeneous networks N^A and N^B are *topologically equivalent* if

- all segments S_i^A of N^A have corresponding segments S_j^B in N^B ($\forall i,j,k$: $S_i^A r_k S_j^A$, $\exists(l,m)$: $S_l^B r_k S_m^B$), and
- at all corresponding intersections, I_i^A and I_j^B, there is the same sequence of adjacent segments in N^A and N^B.

An important component of this assessment is to determine whether all segments in one network have corresponding segments in the other network, and vice-versa. One approach is to perform an exhaustive cyclic permutation of all segments, i.e., trying out all the possible combinations until a match is found. Since the complexity of this method grows exponentially with the number of segments, we use an alternative approach that will eliminate many non-equivalent situations prior to matching corresponding segments in the two networks, therefore, reducing the complexity of the equivalence matching for a large number of situations.

Two homogeneous networks are topologically equivalent if

- they have the same number of line segments (Step 1),
- they have the same number of *#1-meet*, *#2-meet*, and *disjoint* relations (Step 2),
- corresponding segments have the same number of *#1-meet*, *#2-meet*, and *disjoint* relations (Step 3),
- there is a 1:1 association between the segments in each network such that the graphs that form the homogeneous networks are isomorphic (Step 4), and
- at corresponding intersection points in both networks, the adjacent segments have the same orientation (Step 5).

The following example applies this process to assess whether the two homogeneous networks in Figure 10 are topologically equivalent or not.

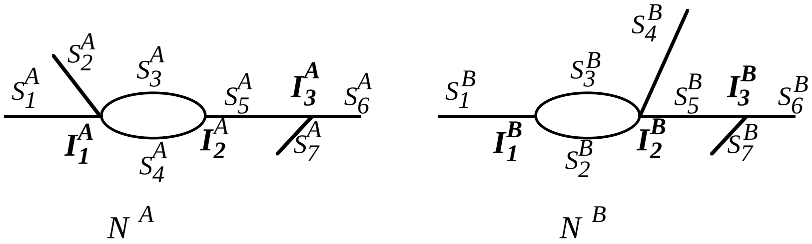

Figure 10: Two homogeneous networks, N^A and N^B.

Step 1: Count the number of segments in N^A (Equation 2a) and N^B (Equation 2b).

number $(S_{i_B}) = 7$ (2a)
number $(S_{j}) = 7$ (2b)

Since both networks have the same number of segments, further properties of the networks must be assessed and compared.

Step 2: Derive from each networks' scene descriptions (Tables 1 and 2) their total numbers of *#1-meet*, *#2-meet* and *disjoint* relations (Equations 3 and 4). The numbers of *#1-meet*, *#2-meet*, and *disjoint* relations are obtained by counting their occurrences in Tables 1 and 2 (Equations 3-4).

	S_1^A	S_2^A	S_3^A	S_4^A	S_5^A	S_6^A	S_7^A
S_1^A		*#1-meet*	*#1-meet*	*#1-meet*	*disjoint*	*disjoint*	*disjoint*
S_2^A			*#1-meet*	*#1-meet*	*disjoint*	*disjoint*	*disjoint*
S_3^A				*#2-meet*	*#1-meet*	*disjoint*	*disjoint*
S_4^A					*#1-meet*	*disjoint*	*disjoint*
S_5^A						*#1-meet*	*#1-meet*
S_6^A							*#1-meet*
S_7^A							

Table 1: The relation matrix of network N^A in Figure 10.

	S_1^B	S_2^B	S_3^B	S_4^B	S_5^B	S_6^B	S_7^B
S_1^B		*#1-meet*	*#1-meet*	*disjoint*	*disjoint*	*disjoint*	*disjoint*
S_2^B			*#2-meet*	*#1-meet*	*#1-meet*	*disjoint*	*disjoint*
S_3^B				*#1-meet*	*#1-meet*	*disjoint*	*disjoint*
S_4^B					*#1-meet*	*disjoint*	*disjoint*
S_5^B						*#1-meet*	*#1-meet*
S_6^B							*#1-meet*
S_7^B							

Table 2: The relation matrix of network N^B in Figure 10.

number (*#1-meet* in N^A) =10 (3a)
number (*#2-meet* in N^A) =1 (3b)
number (*disjoint* in N^A) =10 (3c)
number (*#1-meet* in N^B) =10 (4a)
number (*#2-meet* in N^B) =1 (4b)
number (*disjoint* in N^B) =10 (4c)

Since the counts of each relation type are the same in both networks, a more detailed counting method is used next.

Step 3: For each segment, count the number of *#1d-meet*, *#2-meet*, and *disjoint* relations and summarize how many segments have the same topological relations in N^A (Table 3) and in N^B (Table 4).

	S_1^A	S_2^A	S_3^A	S_4^A	S_5^A	S_6^A	S_7^A	2	2	2	1
#1-meet	3	3	3	3	4	2	2	2	3	3	4
#2-meet	0	0	1	1	0	0	0	0	0	1	0
disjoint	3	3	2	2	2	4	4	4	3	2	2

Table 3: The number of *#1-meet*, *#2-meet* and *disjoint* relations for each segment of network N^A, and the summary counts.

	S_1^B	S_2^B	S_3^B	S_4^B	S_5^B	S_6^B	S_7^B	3	1	2	1
#1-meet	2	3	3	3	5	2	2	2	3	3	5
#2-meet	0	1	1	0	0	0	0	0	0	1	0
disjoint	4	2	2	2	1	4	4	4	2	2	1

Table 4: The number of *#1-meet*, *#2-meet* and *disjoint* relations for each segment of network N^A, and the summary counts.

While the two networks match in one count of segment relations—there are two segments in N^A and N^B that have both three *#1-meet*, one *#2-meet*, and two *disjoint* relations—they differ in all other counts; therefore, the two networks are topologically different and further evaluations (Steps 4-5) are unnecessary.

4. Consistency of Scenes with Heterogeneous Networks

The transition from homogeneous to heterogeneous networks (Section 2.2) involved the addition of regions, and the consideration of their corresponding relations. When assessing topological equivalence for heterogeneous networks, we apply a similar strategy by building on the method developed for homogeneous networks, and extending it by considering the implications from adding regions.

Two heterogeneous networks, N^A and N^B, are topologically equivalent if

- all segments S_i^A in N^A have corresponding segments S_j^B in N^B such that $\forall i,j,k$: $S_i^A r_k S_j^A$, $\exists (l,m)$: $S_l^B r_k S_m^B$;
- all regions R_i^A in N^A have corresponding regions R_j^B in N^B such that $\forall i,j,k$: $R_i^A r_k R_j^A$, $\exists (l,m)$: $R_l^B r_k R_m^B$;
- at all corresponding intersections, I_i^A and I_j^B, there is the same sequence of adjacent segments in N^A and N^B; and
- the boundary sequence (Section 2.2) of each region R_i^A in N^A matches the boundary sequence of the corresponding region R_j^B in N^B.

To test equivalence of heterogeneous networks, we apply the following measures:

- Both networks must have the same number of line segments and regions (Step 1),
- both networks must have the same number of *#1-meet*, *#2-meet*, and *disjoint* relations (Step 2),
- corresponding segments and regions in both networks must have the same number of *#1-meet*, *#2-meet*, and *disjoint* relations (Step 3),
- there must be a 1:1 association between the segments in each network such that the graphs that form the homogeneous networks are isomorphic (Step 4), and
- the intersection orientations and the boundary sequences must be equal at corresponding intersection points and for corresponding regions, respectively (Step 5).

The following example demonstrates how this method is applied to determine whether the two configurations in Figure 11 are topologically equivalent or not.

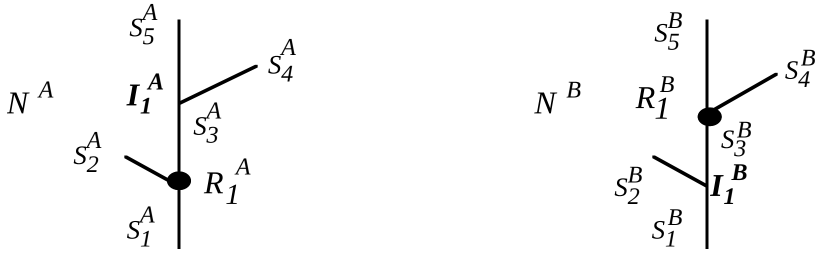

Figure 11: Two heterogeneous networks, N^A and N^B.

Step 1: Determine the number of segments of each network and the number of region in each network.

Since both networks have the same number of segments—count (S_i^A) = count (S_j^B) = 5—and the same number of regions—count (R_i^A) = count (R_j^B) = 1—further comparisons are necessary.

Step 2: Determine the scene description for each network and count the number of *#1-meet*, *#2-meet*, and *disjoint* relations.

The number of the topological relations in the two networks is the same—six *#1-meet*, zero *#2-meet*, and nine *disjoint* relations; therefore, further tests are necessary.

Step 3: Count the number of *#1-meet*, *#2-meet*, and *disjoint* relations for each segment and for each region.

The summary counts show that both networks have the same frequency of the same types of relations—two segments/regions with one *#1-meet* and four *disjoint* relations; two segments/regions with two *#1-meet* and three *disjoint* relations; and two segments/regions with three *#1-meet* and two *disjoint* relations; therefore, one has to perform further analysis.

Step 4: Determine whether the networks' graphs are isomorphic.

The underlying graphs forming the two networks are isomorphic for the mappings shown in Table 5. Given these mappings, the same connectivity can be established in both networks.

$$\begin{array}{ccc}
S_1^A & \equiv & S_5^B \\
S_2^A & \equiv & S_4^B \\
S_3^A & \equiv & S_3^B \\
S_4^A & \equiv & S_2^B \\
S_5^A & \equiv & S_1^B \\
R_1^A & \equiv & R_1^B
\end{array}$$

Table 5: Mappings between the segments and regions in Figure 11 for which the two graphs are isomorphic.

Step 5: Determine the orientation at each intersection, and the boundary sequence along each region, and compare them for corresponding segments (as established in Step 4).

N^A				N^B							
I_1^A	S_5^A	S_4^A	S_3^A	I_1^B	S_1^B	S_2^B	S_3^B	*with*	$S_5^A \equiv S_1^B$	$S_4^A \equiv S_2^B$	$S_3^A \equiv S_3^B$
R_1^A	S_1^A	S_2^A	S_3^A	R_1^B	S_5^B	S_4^B	S_3^B	*with*	$S_1^A \equiv S_5^B$	$S_2^A \equiv S_4^B$	$S_3^A \equiv S_3^B$

Table 6: The orientations around the intersections and along the regions' boundaries for the configurations in Figure 11, together with the mappings of the segments from Table 5.

5. Consistency of Scenes with Homogeneous Networks and Regions

With the establishment of a formalism to determine the equivalence of networks, we turn our attention to the relations between a network and another spatial object. Our goal is to devise formalisms that describe when two configurations with a network and another spatial object have the same topological relations. We pursue this goal in several steps that build upon each other, starting with the most simple configuration—a homogeneous network that has a particular relation with a region—and incrementally increasing the complexity.

To evaluate the topological relation between a homogeneous network N and a region R we describe the network through its segments' relations and then record the topological relations between each segment and the region. Since a homogeneous network consists exclusively of lines, the topological relations of interest are the nineteen line-region relations identified by the 9-intersection [EH91].

Two configurations of a homogeneous network and a region are topologically equivalent if

- the region has the same topological relations to all segments in both configurations,
- these segments have the same topological relations among each other, and
- both regions partition the two networks into equivalent sub-networks.

Figure 12 gives examples of configurations with a homogeneous network and a region that are topologically not equivalent. Figures 12a and 12b differ because the configuration among the region and the part of the network covered by that region are different in both scenes. For the same reason differ Figures 12b and 12c. The configurations in Figures 12a and 12c are topologically different because the line segments covered by the regions do not have the same topological relations with the rest of the network.

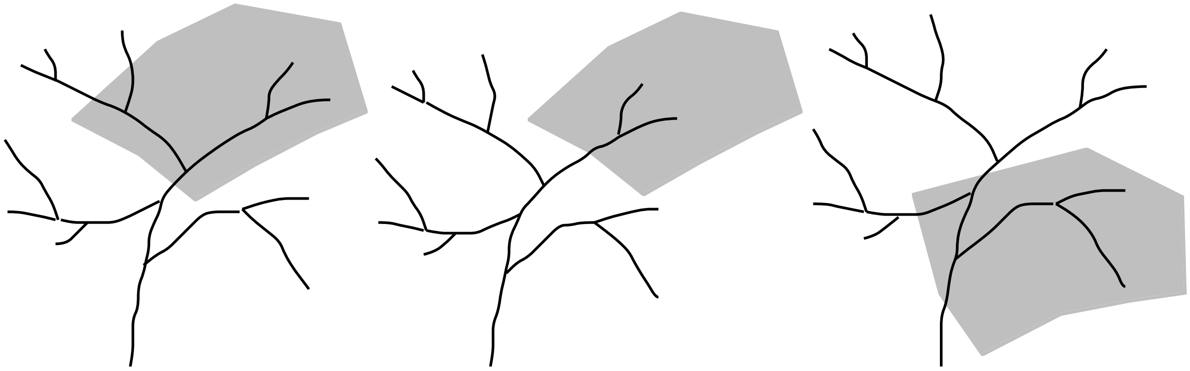
(a) (b) (c)

Figure 12: Three non-equivalent configurations between a homogeneous network and a region.

In order to determine formally whether the topological relations between two homogeneous networks and two regions are the same or not, the following process will be applied:

- $N \cap R^\circ$, $N \cap \partial R$, and $N \cap R^-$ must be the same in N^A and N^B (Step 1),
- the sub-networks $N^A \cap R^A$ and $N^B \cap R^B$ must be topologically equivalent (Step 2),
- the sub-networks $N^A \cap R^{A-}$ and $N^B \cap R^{B-}$ must be topologically equivalent (Step 3),
- the sequence of $N^A \cap R^A$ along ∂R^A must be the same as the sequence of $N^B \cap R^B$ along ∂R^B (Step 4), and
- the sequence of $N^A \cap R^{A-}$ along ∂R^A must be the same as the sequence of $N^B \cap R^{B-}$ along ∂R^B (Step 5).

The following example applies this process to assess whether the two pairs of a homogeneous network and a region displayed in Figure 13 are equivalent or not.

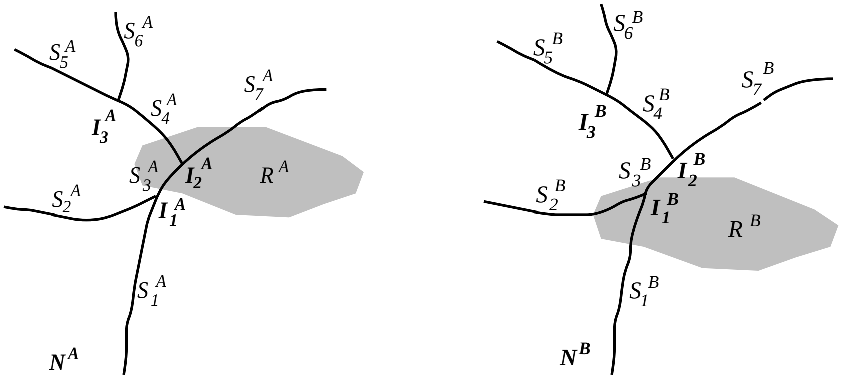

Figure 13: Two scenes with a homogeneous network and a region.

Step 1: Calculate for both networks their intersections with respect to the region's interior, boundary, and exterior.

Since both networks have the same intersections with respect to the regions' parts—they intersect with the region's interior $N^A \cap R^{A\circ} = N^B \cap R^{B\circ} = \neg\varnothing$; the region's boundary $N^A \cap \partial R^A = N^B \cap \partial R^B = \neg\varnothing$; and the region's exterior $N^A \cap R^{A-} = N^B \cap R^{B-} = \neg\varnothing$—further tests must be employed.

Step 2: Determine the sub-networks $N^A \cap R^A$ and $N^B \cap R^B$, and compare their topological structures.

	$S_3^A \cap R^A$	$S_4^A \cap R^A$	$S_7^A \cap R^A$	
$S_3^A \cap R^A$		*#1-meet*	*#1-meet*	*LR 11*
$S_4^A \cap R^A$			*#1-meet*	*LR 11*
$S_7^A \cap R^A$				*LR 11*

Table 7: The relations between the segments parts inside of region R^A.

	$S_1^B \cap R^B$	$S_2^B \cap R^B$	$S_3^B \cap R^B$	
$S_1^B \cap R^B$		*#1-meet*	*#1-meet*	*LR 11*
$S_2^B \cap R^B$			*#1-meet*	*LR 11*
$S_3^B \cap R^B$				*LR 11*

Table 8: The relations between the segments parts inside of region R^B.

	$S_3^A \cap R^A$	$S_4^A \cap R^A$	$S_7^A \cap R^A$	*3*
#1-meet	*2*	*2*	*2*	*2*
LR 11	*1*	*1*	*1*	*1*

Table 9: The summary counts of the relations for each segment inside of network N^A.

	$S_1^B \cap R^B$	$S_2^B \cap R^B$	$S_3^B \cap R^B$	*3*
#1-meet	*2*	*2*	*2*	*2*
LR 11	*1*	*1*	*1*	*1*

Table 10: The summary counts of the relations for each segment inside of network N^B.

Since both sub-networks have the same topological structure—three segments each with two *#1-meet* relations among each other, and one line-region relation *LR 11*—the outer sub-networks must be examined as well.

Step 3: Determine the sub-networks $N^A \cap R^{A-}$ and $N^B \cap R^{B-}$, and compare their topological structures.

	$S_1^A \cap R^{A-}$	$S_2^A \cap R^{A-}$	$S_3^A \cap R^{A-}$	$S_4^A \cap R^{A-}$	$S_5^A \cap R^{A-}$	$S_6^A \cap R^{A-}$	$S_7^A \cap R^{A-}$	R^A
$S_1^A \cap R^{A-}$		*#1-meet*	*#1-meet*	*disjoint*	*disjoint*	*disjoint*	*disjoint*	*LR 2*
$S_2^A \cap R^{A-}$			*#1-meet*	*disjoint*	*disjoint*	*disjoint*	*disjoint*	*LR 2*
$S_3^A \cap R^{A-}$				*disjoint*	*disjoint*	*disjoint*	*disjoint*	*LR 11*
$S_4^A \cap R^{A-}$					*#1-meet*	*#1-meet*	*disjoint*	*LR 11*
$S_5^A \cap R^{A-}$						*#1-meet*	*disjoint*	*LR 2*
$S_6^A \cap R^{A-}$							*disjoint*	*LR 2*
$S_7^A \cap R^{A-}$								*LR 11*

Table 11: The relations between the segments parts outside of region R^A.

	$S_1^B \cap R^{B-}$	$S_2^B \cap R^{B-}$	$S_3^B \cap R^{B-}$	$S_4^B \cap R^{B-}$	$S_5^B \cap R^{B-}$	$S_6^B \cap R^{B-}$	$S_7^B \cap R^{B-}$	
$S_1^B \cap R^{B-}$		*disjoint*	*disjoint*	*disjoint*	*disjoint*	*disjoint*	*disjoint*	*LR 11*
$S_2^B \cap R^{B-}$			*disjoint*	*disjoint*	*disjoint*	*disjoint*	*disjoint*	*LR 11*
$S_3^B \cap R^{B-}$				*#1-meet*	*disjoint*	*disjoint*	*#1-meet*	*LR 11*
$S_4^B \cap R^{B-}$					*#1-meet*	*#1-meet*	*#1-meet*	*LR 2*
$S_5^B \cap R^{B-}$						*#1-meet*	*disjoint*	*LR 2*
$S_6^B \cap R^{B-}$							*disjoint*	*LR 2*
$S_7^B \cap R^{B-}$								*LR 2*

Table 12: The relations between the segments parts outside of region R^B.

	$S_1^A \cap R^{A-}$	$S_2^A \cap R^{A-}$	$S_3^A \cap R^{A-}$	$S_4^A \cap R^{A-}$	$S_5^A \cap R^{A-}$	$S_6^A \cap R^{A-}$	$S_7^A \cap R^{A-}$ *4*	*2*	*1*	
#1-meet	*2*	*2*	*2*	*2*	*2*	*2*	*0*	*2*	*2*	*0*
disjoint	*4*	*4*	*4*	*4*	*4*	*4*	*6*	*4*	*4*	*6*
LR 2	*1*	*1*	*0*	*0*	*1*	*1*	*0*	*1*	*0*	*0*
LR 11	*0*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*1*	*1*

Table 13: The summary counts of the relations for each segment outside of network N^A.

	$S_1^B \cap R^{B-}$	$S_2^B \cap R^{B-}$	$S_3^B \cap R^{B-}$	$S_4^B \cap R^{B-}$	$S_5^B \cap R^{B-}$	$S_6^B \cap R^{B-}$	$S_7^B \cap R^{B-}$	*3* *2*	*1*	*1*	
#1-meet	*0*	*0*	*2*	*4*	*2*	*2*	*2*	*2*	*0*	*4*	*2*
disjoint	*6*	*6*	*4*	*2*	*4*	*4*	*4*	*4*	*6*	*2*	*4*
LR 2	*0*	*0*	*0*	*1*	*1*	*1*	*1*	*1*	*0*	*1*	*0*
LR 11	*1*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*1*

Table 14: The summary counts of the relations for each segment inside of network N^B.

Since the two summary counts are different, no further tests about sequences (Steps 4 and 5) are necessary.

6. Consistency of Scenes with Heterogeneous Networks and Regions

To study this type of configuration we extend the reasoning pattern of the previous section by (1) describing the network through its segments' and regions' relations and (2) recording the topological relations of each of the network's segments and regions with the region of concern.

Two configurations with a heterogeneous network and a region are topologically equivalent if

- the region has the same topological relations to all segments and all regions in both configurations,
- these segments and regions have the same relations among each other, and
- both regions partition the two networks into equivalent sub-networks.

Figure 14 shows an example of two topologically equivalent scenes.

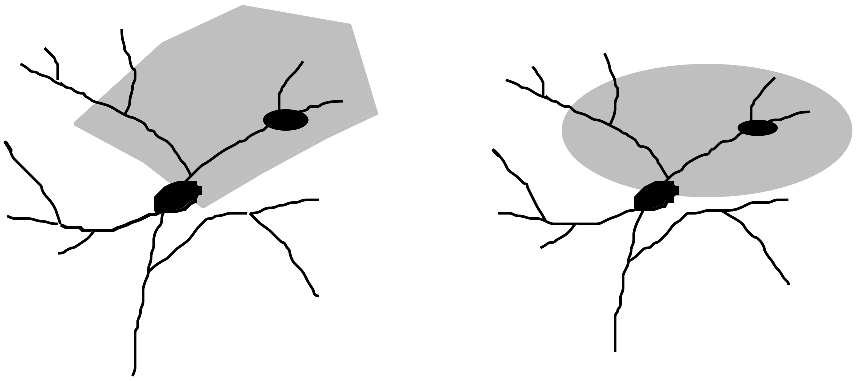

Figure 14: Two topologically equivalent configurations between a heterogeneous network and a region.

The procedure to determine equivalence is the same as for a homogeneous network with a region (Steps 1-5 in Section 4), except that relation matrices (Tables 7, 8, 11, and 12) and summary counts (Tables 9, 10, 13, and 14) also consider the regions that are part of the network. This means that for each region in a network, an additional column exists that captures the corresponding line-region relations.

7. Consistency of Scenes with Heterogeneous Networks and Lines

Configurations with lines, *in lieu* of regions, are covered by a simple modification of the previous model. We modify the reasoning pattern by (1) describing the network through its segments' and regions' relations, and (2) recording the topological relations of each segment and each region with the line, replacing the region from the previous case. Figure 15 depicts a scene of a heterogeneous network and a line.

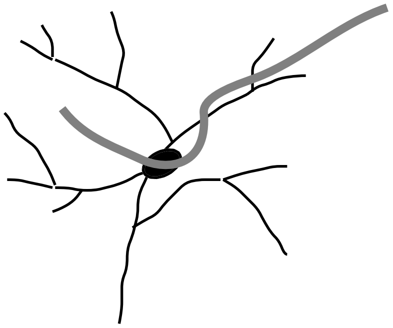

Figure 15: A configuration of a heterogeneous network and a line.

Two configurations are topologically equivalent if

- the line has the same topological relations to all segments and all regions in both configurations,
- these segments and regions have the same relations among each other, and
- both regions partition the two networks into equivalent sub-networks.

The changes in the formalism are straightforward: Everywhere where the two regions R^A and R^B are referred to, the lines L^A and L^B are used. This substitution also implies a change in the types of relations used, namely line-line relations *in lieu* of line-region relations, and region-line relations *in lieu* of region-region relations. The rest of the tests remains the same.

8. Conclusions

We presented a comprehensive method to assess consistency of different scenes involving networks and lines or regions. We assumed that small changes occur between the scenes, so consistency maintenance means topological equivalence among the relations of the objects of the scenes. Particularly, we developed a formalism for topological equivalence of homogeneous and heterogeneous objects, and a formalism for consistency of topological relations with networks. The model is based on the 9-intersection model and uses basic concepts of the component invariants.

We have implemented a consistency checker which evaluates whether the two pairs of a network and a region or line have the same relation, and generate a report of difference. Users construct interactively two pair of objects at the most detailed level, and then draw the corresponding aggregates. The prototype was written in C++ using the MetroWerks code development environment for Macintosh and PowerMacintosh Computers.

An important contribution of the proposed approach is that by looking, at first, simple parameters, such as the number of line segments, it avoids the execution of the brute-force approach when testing topological equivalence. Future research plans lead to the assessment of *similarity* among scenes including roads and urban areas, paths and other linear geographic features. Also the comparison of these results with the help of graph theory is another direction, since scenes' similarity can be detected with the "expensive" heuristic algorithms testing isomorphic graphs, i.e., homeomorphic networks.

Another step towards this direction is to deal with scenes' consistency when large changes occur. Figure 16 illustrates such a case, where network segments of 16a have been dropped or aggregated in 16b. The test algorithms here seem to be much more complicated since the aspect of monotonicity towards the level of less detail (e.g., the number of segments have to be smaller that the one at the more detailed level) leads to a *directed similarity* of the scenes.

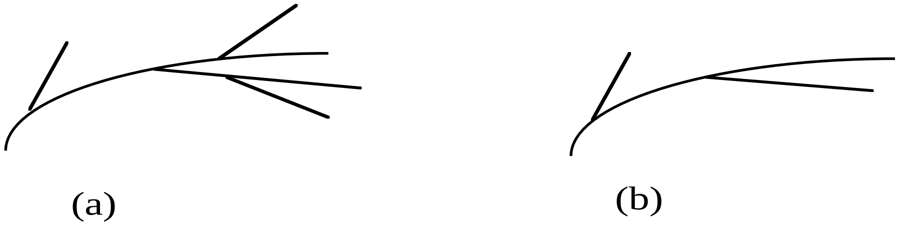

Figure 16: Example of large scale changes.

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