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A Qualitative Spatial Reasoner*

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Abstract

Traditionally, GISs employ purely quantitative methods to represent and infer spatial information. This approach has serious shortcomings when dealing with *qualitative spatial information*, which may be incomplete or imprecise and without knowledge of the particular geometry of the spatial objects involved. This paper describes efforts to build a prototype of a *qualitative spatial reasoner* about spatial relations such as topological relations, cardinal directions, and approximate distances. It builds on relation algebras developed for the individual spatial relations. The system is extensible as demonstrated by the inclusion of temporal relations. The novel concept in this object-oriented setting is the treatment of relations as *first-class objects*, rather than as labeled links between spatial objects.

1. Introduction

The paradigm of "complete and exact spatial information" underlies the use of any current commercial Geographic Information System (GIS); therefore, current GISs are very good at integrating and analyzing *quantitative* spatial information, dealing with spatial data modeled in a Cartesian coordinate space. While quantitative representations allow for very powerful and frequently efficient calculations, they fall short when users lack some information about the geometry of the objects involved. Quantitative representations always need a complete description of the objects' geometry, i.e., they cannot handle partial information, and they have serious problems when geometric information is imprecise. Problems also arise due to finite-precision computations and the resultant error propagation.

However, scenarios with incomplete, imprecise, and qualitative spatial information occur frequently when users want to analyze spatial descriptions. Narrative, as in newspaper articles, trip descriptions, and emergency reports (Welebny 1993), include descriptions of geographic space without the required precise description of the objects involved needed in order to allow a GIS to fill the gaps and infer missing information. Similarly, instructions humans give to guide others through geographic space, contain combinations of spatial descriptions without references to coordinates (McGranaghan *et al.* 1987; Golledge 1992). In another domain, biologists collected herbarium specimens, for which they recorded

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narrative descriptions of the sites where each specimen was found (Futch *et al.* 1992). Automated spatial analyses, for instance, about endangered species or the relations to soil types and climate, are severely hampered by the lack of methods to integrate the individual natural language descriptions of geographic spaces to infer and compare the spatial relations among the specimens. In all cases, a presently available commercial GIS would require a user to identify the location of the object and geocode a complete description of the object's geometry, most often in terms of points, lines, and areas and their inter-relationships.

It has been recognized that a quantitative approach is an inappropriate representation of human cognition and spatial reasoning (Kuipers 1978). The remoteness from familiar or intuitive processes makes Euclidean geometry reasoning systems inappropriate for applications with a high level of user interaction, since they deal with different concepts—small set of symbols on an ordinal and nominal scale in a discrete space vs. quantitative calculations in an infinitely precise, continuous space—which have significantly different properties. Analytical geometry and Cartesian coordinates have also been found as inappropriate tools, e.g., for the integrating the biologists' narrative descriptions of geographic locations.

This paper pursues a radically different approach to handling geographic data. Rather than forcing a user to translate all spatial concepts into a quantitative framework, users can reason about *qualitative spatial information* within a purely qualitative environment. While quantitative models use absolute values, *qualitative models* deal with magnitudes. The advantage of qualitative reasoning models is that they can separate numerical analyses from the determination of magnitudes or events, which may be assessed differently depending on the context. In qualitative reasoning a situation is characterized by variables that can only take a small, predetermined number of values and the inference rules that use these values *in lieu* of numerical quantities that approximate them (de Kleer and Brown 1984).

Qualitative information and qualitative reasoning are not seen as substitutes for quantitative approaches, rather they are complementary methods, which should be applied whenever appropriate. Quantitative spatial relations include such observations as bearings (150° 25'), distances (4.3 miles), and corresponding values derived from coordinates. Such quantitative values are in close relationship with some qualitative spatial relations (Hong 1994). For example, if the azimuth to a point (measured clockwise from due north) is 90° then this corresponds to the cardinal direction east. Likewise, if two regions *meet* topologically, then the distance between their boundaries is 0. For many decision processes qualitative information is sufficient; however, occasionally quantitative measures, dealing with precise numerical values, may be necessary, which would require the integration of quantitative information with qualitative reasoning. Qualitative approaches allow the users to abstract from the myriad of details by establishing "landmarks" (Gelsey and McDermott 1990) when "something interesting happens;" therefore, they allow them to concentrate on a few but significant events or changes (Egenhofer and Al-Taha 1992). This working pattern is typical for scientists and relevant for geographic databases in which scientists record the data of their experiments—frequently time series observations—with the goal of subsequently extracting the "interesting" stages.

Our investigations focus on large-scale geographic space, which is defined as space that is beyond the human body and cannot be observed from any single viewpoint (Kuipers 1978;

Kuipers and Levitt 1988). Unlike small-scale or table-top space as used in the context of qualitative kinematics or mechanical parts, geographic space is commonly subject to incomplete and imprecise information for human spatial reasoning; therefore, in the absence of more precise geographic information, purely qualitative geographic reasoning may be used as a substitute. While these reasoning processes may provide only approximate, sometimes crude solutions, they are frequently the only means available to infer new information that may still be sufficient to solve a particular geographic problem.

The remainder of this paper introduces the concepts of a spatial reasoner about qualitative spatial relations and describes a prototype implementation. Section 2 discusses the basic reasoning mechanisms employed for qualitative spatial reasoning. Section 3 reviews the formalisms used for topological relations, approximate distances, cardinal directions, and temporal relations. Section 4 describes the conceptual design and architecture of the spatial reasoner. In Section 5, a prototype implementation is presented and Section 6 concludes with a discussion of future work.

2. Spatial Reasoning about Qualitative Relations

For a qualitative spatial reasoner, it is important to find representations that support partial and imprecise information. Euclidean geometry is not a good candidate since it relies on the existence of complete coordinate n-tuples. Likewise, pictorial representations are inadequate since they overdetermine certain situations, for example, when drawing a picture representing a cardinal direction, a sketch also includes information about the sizes of the objects and some relative distances. The additional information has the potential for inconsistencies and ambiguities primarily due to the fact that graphical representations often have no basis in any formalism.

This qualitative spatial reasoner is based on the representation of *explicit spatial relations*. It allows users to record spatial-relation information independent of the actual geometry of the spatial objects. Examples of explicit spatial relations are *cardinal directions* such as north or north-east; *approximate distances* such as near and far; and *topological relations* such as inside, disjoint, or overlap. Furthermore, *temporal relations* will be considered to describe spatial objects at different states in time. The qualitative spatial reasoner supports queries of the following kind:

- What is the spatial relation of type *x* between the given spatial objects (where *x* may be “topological,” “cardinal direction,” “approximate distance,” or “temporal”)?
- What are all known spatial relations (of any type) between the given spatial objects?
- Which objects have a particular spatial relation with a given spatial object?
- Which objects are linked by a particular spatial relation?
- Are two given spatial objects linked by a particular spatial relation?

Unlike Euclidean-based geographic databases and GISs, which provide for a single way of determining such relations by quantitative calculations, the spatial reasoner can infer information at different conceptual levels. In geographic applications entities are defined both in terms of their attributes and the complex spatial relations, such as proximity and connectivity, between them. This spatial structure is of primary interest in geographic databases and hence must be represented or modeled in some form. The two approaches are (1) to define it explicitly in a relational form or (2) to construct it using rules in a deductive database. We combine these approaches by defining relations as first-class objects, which have an associated spatial reasoning system; therefore, such a spatial reasoner has three basic approaches to determining the result of a geographic database query:

- use explicitly stored, qualitative information if it meets the requirements;
- infer the result with qualitative reasoning formalisms; and
- compute a qualitative spatial relation by transforming the query into a quantitative Euclidean coordinate space, in which the problem gets solved by applying algorithms on some model such as a raster or vector representation, and map the quantitative result back onto a qualitative value.

2.1 Explicitly Stored Relations

In the most simple situation, the spatial relation for which a user asks is explicitly stored in the database; therefore, a particular kind of spatial relation can be retrieved immediately and query processing becomes a simple table look up. For example, if the cardinal direction between Bangor and Orono is recorded, then there is no need to compute it.

2.2 Qualitatively Inferred Relations

The behavior of spatial and temporal relations is captured in relation algebras. These algebras formalize particular properties of relations that are crucial when deriving information.

- A relation r is symmetric if $A\ r\ B$ implies $B\ r\ A$.
- Two relations r_1 and r_2 are converse if $A\ r_1\ B$ implies $B\ r_2\ A$.

The most powerful property is the *composition*. It infers the relation r_1 between two objects A and C from the knowledge of the relations $A\ r_2\ B$ and $B\ r_3\ C$.

Frequently, the inferred relation is imprecise and represented by a disjunction of several possible relations. An important property of composition is that it distributes over disjunctions, i.e.,

$$r_i \,;\, (r_j \vee r_k) = (r_i \,;\, r_j) \vee (r_i \,;\, r_k) \qquad (1)$$

The transitivity of a relation is a special case of the composition in which $r_1 = r_2 = r_3$ such that $A\ r\ B$ and $B\ r\ C$ implies $A\ r\ C$.

A constraint network is used for representing the relations and for evaluating consistency or inferring relations. The nodes in the network represent individual objects, while each arc is labeled by the possible relations between the two objects at its nodes. Consistency is maintained by computing all consequences whenever a new relation is added to the network. The consequent relations are determined by computing the transitive closure of the relations using the appropriate transitivity table.

Such qualitative inferences allow a query processor to derive answers even if the particular spatial or temporal relation has not been recorded explicitly. For example, if a system stores that “Orono is north of Bangor” and a user asks the query for the cardinal direction between Bangor and Orono, given the knowledge that north and south are converse relations, the query processor can infer that Bangor is south of Orono.

2.3 Quantitatively Calculated Relations

Quantitative evaluation of spatial relations is the most commonly used method in commercial GISs. The geometry of the objects is represented in terms of a set of coordinates or pixels in some Cartesian reference grid. Computational geometry algorithms such as nearest neighbor search, point in polygon tests, and line-line intersections are used to compute spatial relations like inside, within X miles, or north of.

It is important to note that these spatial computations are performed on a model and a necessarily finite precision representation of the reality of interest. The results of the computations are then mapped onto spatial relations between the real world entities. The underlying assumption here is that the model and its representation are an accurate and valid encapsulation of the nature and properties of the geographic entities.

3. Formalisms for Qualitative Reasoning

Qualitative spatial relations have been the subject of extensive research over the last years and numerous formalisms and prototype systems exist (Dutta 1989; Mukerjee and Joe 1990; Freksa 1992; Cui *et al.* 1993; Hernández 1993; Papadias and Sellis 1993). This work builds on formalisms for reasoning on interval-based temporal relations (Allen 1983), topological relations (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991), and qualitative reasoning about distances and directions (Frank 1992, Hong 1994) in the design and development of an object-oriented spatial reasoner. These formalisms could be substituted or complemented by any other formalization of spatial relations that follows the guidelines of a relation algebra (Tarski 1941).

3.1 Topological Relations

The relation algebra for topological relations is based on the 4-intersection, which analyzes the intersections of the two objects’ boundaries (∂) and interiors ($^\circ$) (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991). We illustrate the use of this algebra with the theory developed for region objects (Figure 1). The same method is applicable to topological relations between any combination of regions, lines, and point by using the 9-intersection, which includes also the intersections with the objects’ exteriors (Egenhofer and Herring 1991).

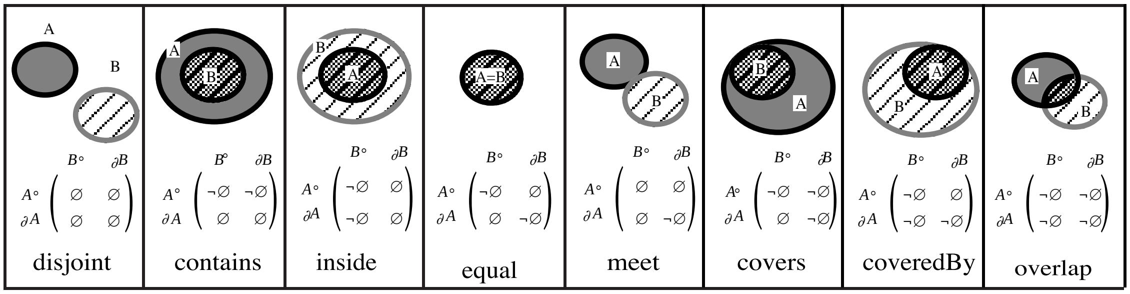
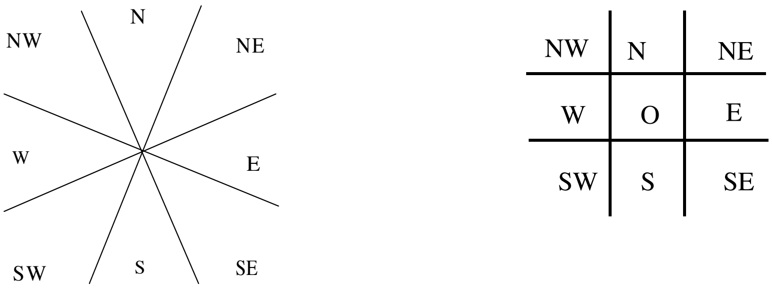


Figure 1: Examples of the 8 topological relations between regions in $\mathbb{R}^2$.

The basis of the reasoning mechanism is the *composition* of topological relations, which is defined by the composition table. The composition table was formally derived based on the transitivity of empty and non-empty intersections and gives the results of the 64 compositions of the 8 basic region-region relations (Egenhofer 1991).The composition of two topological relations, denoted by $r_1 ; r_2$, could result in a single topological relation e.g., *meet* ; *contains* = *disjoint*, or more than one, e.g., *contains* ; *meet* = *contains* $\vee$ *covers* $\vee$ *overlap*. This enables the problem of evaluating topological consistency and inferring relations to be expressed as constraint satisfaction problems (Egenhofer and Sharma 1992; Smith and Park 1992; Egenhofer and Sharma 1993).

3.2 Cardinal Directions

Unlike topological relations, cardinal directions could be quantitative values, such as azimuth or bearing, or qualitative symbols, such as north or north-east. The choice of measure depends on the application. The spatial reasoner uses both azimuth and cardinal directions and the choice is made based on the specified query constraints and availability of information.



(a) (b)

Figure 2: (a) Cone-shaped (Peuquet and Ci-Xiang 1987) and (b) projection-based systems with neutral zone (Frank 1992) for cardinal directions.

The prototypical concept of cardinal directions comes from the compass, from which the idea of cone-shaped areas each associated with a specific direction has been derived (Figure 2a). An equally useful construction is based on projections (Figure 2b), where the directions are defined by half-planes. Similar to topological relations as formalized by the 9-intersection, one can construct relation algebras for the different models of cardinal directions (Frank 1992).

3.3 Approximate Distances

Distances are quantitative values determined through measurements or calculated from known coordinates of the two objects in some reference system. Humans, however, frequently use approximations and qualitative notions such as near or far when reasoning about distances. Such qualitative distances are defined by a set of symbols, denoting qualitative measures such as *near* or *far*, and addition rules—sometimes in combinations with direction reasoning—such as *near* plus *far* is *far* (Frank 1992). Approximate distances are mapped onto quantitative distances using fuzzy sets (Dutta 1989) or mutually exclusive distance intervals of increasing ratio (Hong 1994). Reasoning with approximate distances, however, only provides meaningful results in conjunction with direction reasoning.

3.4 Temporal Relations

The concept of the qualitative spatial reasoner extends to other relations with a qualitative relation algebra. For example, temporal changes of different spatial objects may be modeled by Allen's (1983) popular temporal logic, which is based on intervals. Intervals are defined as a set of points along a discrete time line between a specified start and end point. Any ordered pair of intervals will be related in one of thirteen ways (Figure 3).

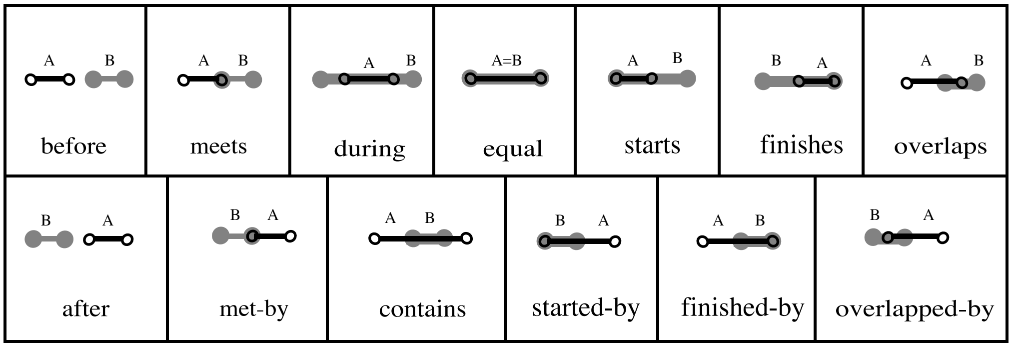


Figure 3: Examples of Allen's thirteen 1-dimensional interval relations.

Again, a relation algebra describes the properties of the relations and the results of the compositions of temporal relations are captured in the composition table. This algebra for the relations defines a set of rules that permit inferences about temporal relations.

4. Object-Oriented Design for a Qualitative Spatial Reasoner

The conceptual design of our qualitative spatial reasoner differs from conventional GISs and spatial reasoning systems by treating relations as explicit *objects*, rather than labeled *links* between spatial objects. This looser framework permits the system to be used as a test bed for qualitative reasoning, which can be expanded to accommodate additional types of (spatial) relations. It leads to an object-oriented implementation in which objects as well as relations have operations and respond to messages.

Figure 4 shows the object hierarchy developed for the reasoner. There are two first-order classes in this hierarchy, the **Relation** and the **Relative**. The **Relative** is the abstract class for objects involved in **Relations**. It provides methods for getting access to the **Relations** in which a **Relative** is involved, getting its name, and for dissolving **Relations** when the **Relative** is deleted. All **Relatives** require a name, which is used as an identifier.

Either a **Place** or an **Event** may participate in a relation. For example, the place Bangor is South of Orono, and the event Graduation Ceremony occurred after Final Exams. Thus a **Place** is a **Relative** with a location while an **Event** is a temporal occurrence. The location was defined as a 3-dimensional Cartesian coordinate. Much of the reasoning over **Places** assumes a representative coordinate for objects that have, at least, a two-dimensional extent. This ambiguous representation is very similar to human reasoning over hierarchically ordered spaces (Frank 1992), but can produce erroneous results on occasion. For instance, when you are in Reno Nevada, California is to the northwest, west, southwest, south and southeast, but the result calculated from the representative points would be south or even southeast.

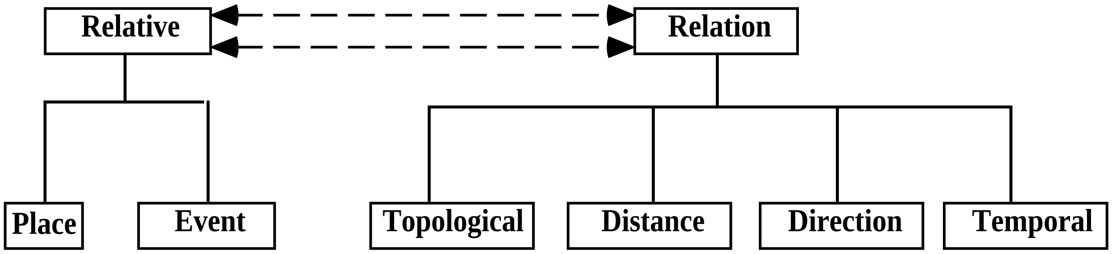


Figure 4: Object hierarchy for a qualitative geographic reasoner.

Each **Event** is a discrete object that may have a starting time and ending time. This model does not permit temporal objects that have intermittent or periodic activations such as street lights being lit only at night.

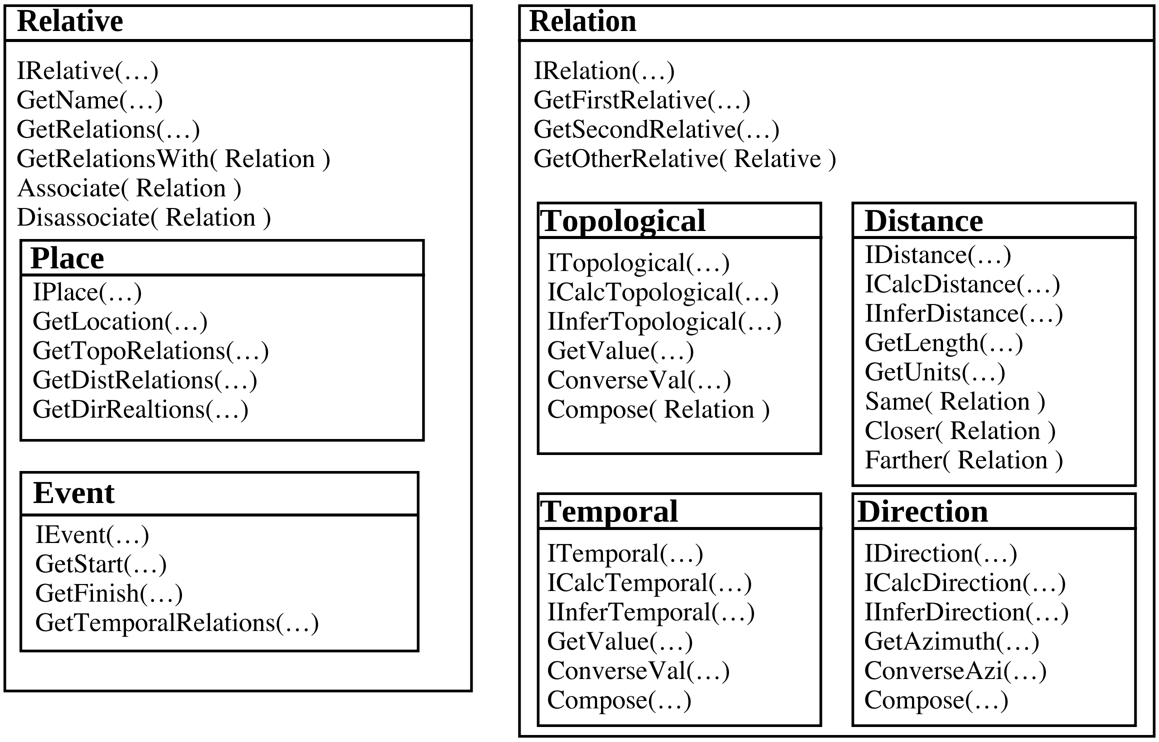


Figure 5: Methods for **Relative** and **Relation** classes and their subclasses.

The **Relation** class is an abstract class that binds two **Relatives** in a spatial or temporal relation. It provides basic utilities for relation management such as retrieving the **Relatives** involved, and creating and destroying the **Relation**. Each of the types of relations described in the previous section is a subclass of **Relation**. It has its methods for accessing an object's value, e.g., the value *overlaps* in a **Topological Relation** (Figure 5). Again it should be noted that the **Relation** has a value appropriate to its type, and limited reasoning can be accomplished without reference to the values stored by the **Relatives** involved in the **Relation**.

Each class provides three separate initializers for explicit, calculated, and inferred creation.

- Explicit creation is used when a fact is known about two **Relatives**, for example Edinburgh *is north of* London. Neither of the relatives referenced in the relation need to have exact coordinates to be part of an explicit definition. This is similar to receiving a news report that there was rioting in South Central Los Angeles.
- Calculated creation depends on both **Relatives** being completely defined quantitatively. This is the way most GISs currently treat topology definition and, therefore, complete and accurate geometric definition is assumed on the part of users of the system.
- Inferred creation is used when a **Relation** is needed between two **Relatives**, but the relationship is not known explicitly and the **Relatives** are not quantitatively defined. It is possible that such a request will fail, meaning that no unambiguous relation could be derived at this time.

5. A Prototype Qualitative Spatial Reasoner

The prototype spatial reasoner was developed using Symantec Think C and the Think Class Library[1]. This environment provided a framework for prototyping the user interface using standard Apple Macintosh graphic user interface elements. Two additional object classes were required to implement the prototype to permit persistent storage of **Relations** and **Relatives** in the spatial reasoner:

- The **Persistent** class provides methods for maintaining references to instances of objects regardless of whether they are currently in memory or on disk. This permits a much more efficient use of memory and avoids having to reload the entire database into active memory before analysis can begin. Both **Relatives** and **Relations** are implemented as subclasses of the **Persistent** class. Persistent objects are stored in a **Home** object. In this implementation, there is a single **Home** for all data. It would certainly be feasible to also use an object-oriented or an extensible relational database management system as the persistent store.
- A second important element of the implementation was the mechanisms for building and querying the database. Figure 6 shows the input screen for new **Relatives**. In this case, an **Event** is being added to the database. The city of Bangor, Maine was founded in 1724, but we have no definite ending date, yet. However, the only required field is the name; therefore, this **Relative** can be added to the database.

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[1] Code is available via anonymous FTP from mecan1.maine.edu as the file spatial-reasoner.hqx.sit in the directory pub/NCGIA/Software.

Type: Event
Name: Bangor, Maine
Start: 1724
Finish:

Figure 6: **New Relative** entry screen.

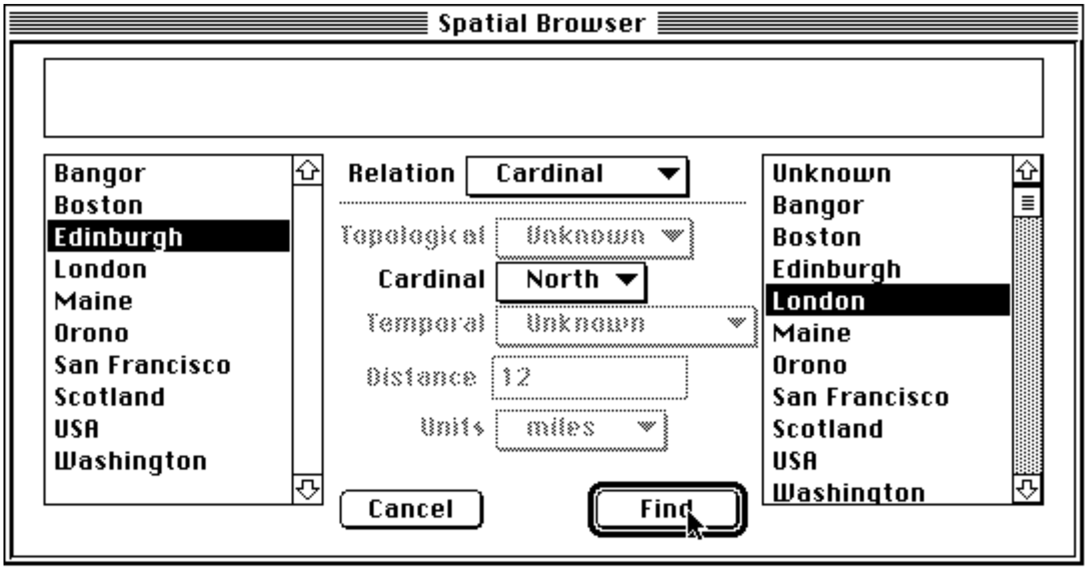
The query processor provides means for answering the following general questions:

- "What **Relations** exist between two specific **Relatives**?"
- "What other **Relative** has the specified **Relation** to a given **Relative**?"
- "Does the specified **Relation** exist between two specified **Relatives**?"

Currently, the user is restricted from asking the question, "What are all the **Relations** between a given **Relative** and all other **Relatives**?"

The user must pick a **Relative** from the list on the left side of the Spatial-Temporal Browser and then may pick a class of **Relations** and the specific **Relation** value desired. If the **Relation** is unknown, the category "All" may be selected instead of a specific class of **Relations**. Finally a **Relative** from the right hand list is selected. The right hand list also includes the value "Unknown," if the second **Relative** is the objective of the search. The Browser composes the query and then sends the **Home** a "RelationExists" message.

The **Home** is responsible for processing the query and determining whether it can find the **Relation** between the two **Relatives** in its database. The query is first processed to find explicit **Relations** stored. If none can be found, the **Home** attempts to calculate the **Relation** from information that may be stored by each **Relative** in the search. Should that fail, the **Home** tries to infer the answer from its stored data, either by finding the converse **Relation** or composing the **Relation** from other stored information. Results are displayed in a text window as descriptive statements about the **Relations** found (Figure 7).



Relation: Cardinal
Topological: Unknown
Cardinal: North
Temporal: Unknown
Distance: 12
Units: miles

Figure 7: A user-interface snapshot of the Spatial Browser.

6. Conclusions

This paper described efforts in building a qualitative spatial reasoner around individual relation algebras for topological relations, approximate distances, cardinal directions, and temporal relations. It demonstrated that such a system can effectively be constructed using an object-oriented design methodology and viewing spatial relations as objects in themselves. The prototype developed served as a proof of concept that underscored the utility of the approach.

A number of theoretical problems need to be solved before such a qualitative reasoner can become operationally useful and viable. Some of the issues are:

- Formalizations are needed to describe cardinal directions and approximate distances among *extended* objects. Currently available relation algebras for qualitative distances and directions only apply to point objects. This simplification, however, is incompatible with the more interesting and non-trivial cases of topological relations between areal and linear objects.
- With a set of homogeneous spatial-relation algebras, inferences among different types of spatial relations (i.e., compositions across relations) can be tackled. Compositions of different kinds of relations may have meaningful results, e.g., *A north B*; *B contains C* implies (*A north C* and *A disjoint C*). Such inferences cannot be made from isolated reasoning about the individual relations.
- In large databases, qualitative inference methods may be unacceptably expensive computationally. There is a need for devising strategies that limit the search space for qualitative inferences (e.g., hierarchical systems).
- Problems due to the differences in the use and semantics of natural language spatial predicates must be resolved, otherwise a qualitative reasoner based on a certain set of

spatial concepts or semantics would be non-intuitive for users with a different set of concepts.

- In order to present query results in a graphical form different from the symbolic (alphanumeric) values, it is necessary to generate graphical presentations for qualitative spatial information. Such figures may be only sketches, sometimes highly distorted, and yet sufficiently informative.

Successful solutions in these areas will contribute to the design and development of an integrated spatial reasoner that complements and enhances a Geographic Information System.

Acknowledgments

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