Evaluating Inconsistencies Among Multiple Representations*

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Abstract

If Geographic Information Systems (GISs) contain multiple representations of the same geographic objects at different levels of detail, it becomes necessary to compare the different representations and assess whether they contradict each other or not. Topological information is generally considered first-class geographic information and as such the preservation of topological relations among objects in different representations manifests a critical criterion for the comparison of multiple representations and their consistency evaluation. This paper describes a framework within which the topological consistency of multiple representations can be assessed. The rational for assessing topological similarity is the *monotonicity assumption* of a generalization, under which the topology of any object and any topological relation between objects must stay the same through consecutive representation levels; or continuously decrease in complexity and detail. Such changes are assessed through object similarity and relation similarity, respectively. Within this framework, only those topological invariants can be changed that are at least on an ordinal scale.

1. Introduction

"Multiple representations encompass changes in geometric and topological structure of a digital object that may occur with the changing resolution at which that object is encoded for computer storage, analysis, and depiction" (Buttenfield 1989). The concept of multiple representations is a matter of redundancy. The same objects are represented in several different ways, tailored towards the needs of different users or analysis operations. Typical examples of applications that require multiple representation levels are cartographic map series at different scales and car navigation systems that allow users to plan trips and give them driving instructions. In order to exploit the information across the different representations, it is important that certain aspects of the geometry of the objects get

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captured consistently throughout the different representations. Although research has been undertaken in recent years on various aspects of multiple representations such as data models (Bruegger and Frank 1989; Bruegger and Kuhn 1991; Kilpeläinen 1992; Timpf *et al.* 1992) and cartographic generalization (Buttenfield and McMaster 1991), we lack currently methods to assess consistency among different representations and to maintain consistently multiple representations of geographic objects.

The issue of consistency across multiple representations is of primary importance for the successful adoption of multiple-representation GISs. *Consistency* refers to the lack of any logical contradictions within a model of reality. This must not be confused with *correctness*, which excludes any contradiction with reality. An *inconsistent* data set violates certain consistency constraints. Inconsistencies in an information system lead to erratic behavior as queries that a user expects to generate the same result, may produce different results. Users are irritated by such behavior and it may call into question the results of any query, the end result being that a frustrated user will give up this system as it gives the impression of being an unreliable assistant.

Conceptually, a multi-representation GIS may be considered as a set of several separate GISs, each of which covers the same geographic area and corresponds to a different level of detail. In itself, each individual level may be consistent, however, when integrating and comparing the different levels, inconsistencies may be detected if the representations contradict. This paper will focus particularly on the *consistent modeling of spatial relations among several objects*, each represented at multiple levels of details, and the *topological consistency constraints* that must hold among the different representations of objects. The individual representations are assumed to be topologically consistent (Egenhofer and Sharma 1992).

Our investigations are led by the hypothesis of *dominance of topological relations.* We start from the observation that human spatial reasoning uses the topological structure of space and the organization of space induced by the objects in it. As such, we are primarily concerned with the relations among objects embedded in space, and care less about the representations of the objects. This allows us to focus fully on topological properties. In this context, topological information is considered first-class information, which has to prevail in the case of a conflict between two different representations. The dominance of topological relations corresponds with observations by some researchers in map generalization who acknowledge that the violation of topological relations is the major source of conflicts among generalizations (Zhang and Mark 1993).

Our approach to dealing with consistency in multiple representations is novel. In the past, most related work in multiple representations has focused on cartographic line generalization (Buttenfield and McMaster 1991) and algorithms to derive a coarser line from a line with more detail (Douglas and Peucker 1973; Muller 1990; Beard 1991b; Wang and Muller 1993). Topological consistency is usually treated at the lower level of data structures, counting the number of nodes and arcs to assure that an object's topology is complete (Laurini and Thompson 1992). This method is appropriate if only an object's metric changes and one wants to evaluate whether its topological structure has been preserved; however, it does not capture relations among objects and sometimes necessary changes in the topology such as a change in the dimension, the aggregation of several parts into a single objects, or the elimination of holes. Likewise, the work on hierarchies of cells (Bruegger and Frank 1989; Bruegger and Kuhn 1991) addresses topological and metric dependencies across different levels of resolution at the level of the representation of the objects.

Some issues are excluded from the present discussions: (1) All objects at all levels of detail are assumed to be represented in the same spatial data model. This eliminates a facet of multiple representations in which the same object is represented by different spatial data models such as raster and vector (Peuquet 1988). Some aspects of this discussion have been treated under the consistent mapping of topological relations between vector and raster data (Egenhofer and Sharma 1993). (2) Only topological changes will be considered, while any metric changes that do not imply a change in the object's topological relations with respect to other objects will be disregarded. (3) We are not concerned with the operations of how to *derive* one representation level from another. This is addressed by other researchers in efforts to develop sets of generalization operations (Beard 1991a; Mackaness 1991; McMaster and Shea 1992). (4) No display issues are considered.

The remainder of this paper is structured as follows: Section 2 briefly reviews the model used to describe topological relations. Section 3 introduces the notations and formalisms that will be used for multiple representations. Formalism to assess object similarity and relation similarity are presented in sections 4 and 5, respectively. Finally, the conclusions in section 6.

2. Topological Relations

The model for topological relations used for this project extends the result of previous investigations (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991) and characterizes the topological relation t between two point sets, A and B, by the set intersections of A's boundary (∂A) and interior (A°) with the boundary and interior of B, called the *4-intersection* (Equation 1).

$$I(A,B) = \begin{pmatrix} \partial A \cap \partial B & \partial A \cap B^\circ \\ A^\circ \cap \partial B & A^\circ \cap B^\circ \end{pmatrix} \tag{1}$$

2.1 Content Invariant

With each of these four intersections being empty ($\varnothing$) or non-empty ($\neg\varnothing$), the model has 16 possible topological relations between two point sets, some of which cannot be realized. For two simple regions without holes, the categorization shows 8 distinct topological relations. They have been called *disjoint*, *meet*, *equal*, *overlap*, *inside*, *contains*, *covers*, and *coveredBy* (Figure 1).

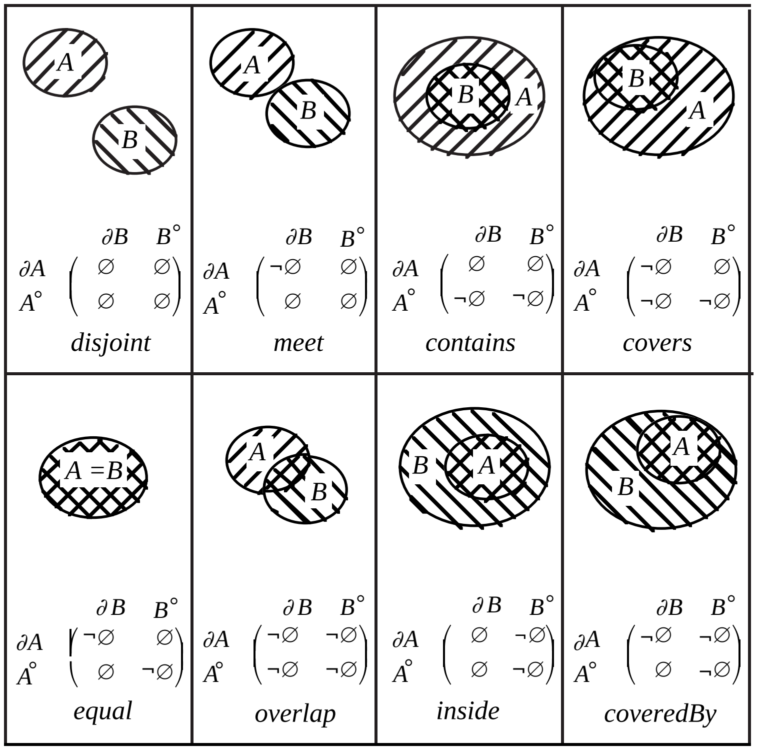


Figure 1: The eight topological relations that can be realized between two spatial regions.

2.2 Component Invariants

More detailed distinctions are possible if further criteria are employed to evaluate the non-empty intersections. In order to establish *topological relation equivalence* (i.e., to decide whether or not two pairs of objects have the same topological relations), it is sufficient to describe such invariants for the components (or separations) of the boundary-boundary intersection only, since the other three intersections can be inferred from them (Egenhofer and Franzosa 1993). The necessary invariants to consider are:

- The *sequence* of components counted along the boundaries. Based on an orientation of the plane, one follows the boundary of one of the two regions and numbers progressively the components in the intersection with the boundary of the other region. One records the sequence of components and their topological invariants, as they are visited when walking along the boundary of the other region.
- The *dimension* of components: for boundary-boundary components it can be either 0 or 1, depending on whether the boundaries intersect in a point or a line, respectively.
- The *type* of boundary-boundary component intersection. The components either *touch* (t) if the boundary enters and leaves the intersection from the same part, or *cross* (c) if the boundary enters from a different part than it leaves. *Crossing* components only apply for regions that *overlap*, while *touching* components may occur with *overlap*, *meet*, *covers*, and *coveredBy* relations. *Crossing* may be further refined depending on whether the component *crosses into* (*cI*) or *crosses out* of (*cO*) the other region's interior. Finally, a 1-dimensional *cross* can be refined into an *inner cross* (*ic*) if the

component is contained in the interior of $A \cup B$, or an *outer cross* (*oc*) if it is in the boundary of $A \cup B$.

- The *boundedness*, i.e., whether a component is next to a bounded or unbounded exterior, can be distinguished by calculating for each component C_i of the boundary-boundary intersection the following set: $\partial((A \cup B)^*) \cap C_i$. If it is empty then C_i is *bounded* (*b*), otherwise it is *unbounded* (*u*).

Detailed topological relations between two regions will be expressed by the *4-intersection matrix M(A, B)*, which describes the content invariant, and in the case of non-empty boundary-boundary intersections the *component invariant table* $T(\partial A, \partial B)$ listing the sequence of boundary-boundary components and each component's dimension, type, and boundedness. For example, the *overlap* configuration in Figure 2 is described by its 4-intersection matrix (Equation 2) and the component invariant table (Equation 3).

$$M(A,B) = \begin{pmatrix} \neg\emptyset & \neg\emptyset \\ \neg\emptyset & \neg\emptyset \end{pmatrix} \tag{2}$$

$$T(\partial A, \partial B) = \left[\begin{array}{c|ccccccc} \text{sequence} & 1 & 5 & 6 & 7 & 4 & 3 & 2 \\ \hline \text{dimension} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{type} & \text{icO} & \text{cI} & \text{cO} & \text{cI} & \text{cO} & \text{t} & \text{cI} \\ \text{boundedness} & \text{u} & \text{u} & \text{u} & \text{u} & \text{b} & \text{b} & \text{b} \end{array} \right] \tag{3}$$

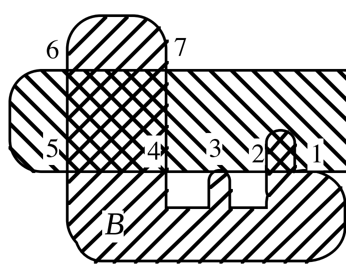


Figure 2: A detailed topological relation.

3. A Multiple-Representation Framework

The framework within which the topological consistency of multiple representations will be considered consists of a set of *representation levels*, $S_0 \ldots S_n$. Two different levels, S_i and S_{i+1}, are linked through some generalization operations. In cartography, these generalizations are generally considered complex transformations and may include simplification, smoothing, aggregation, amalgamation, merging, collapse, refinement, exaggeration, enhancement, and displacement (McMaster and Shea 1992). Each transformation reduces the complexity of a representation level and generates a representation level that is at most as general as the original representation level.

The generalization operations are governed by the *monotonicity assumption*:

- *The changes of topology through consecutive representation levels must be monotonic.*

Therefore, representation levels are ordered by the relation "topologically less general than or topologically as general as," denoted by $S_i \leq S_{i+1}$, which is reflexive, antisymmetric, and transitive (Equations 4a-c).

$$S_i \leq S_i \tag{4a}$$

$$(S_i \leq S_j) \wedge (S_j \leq S_i) \Rightarrow S_i = S_j \tag{4b}$$

$$(S_i \leq S_j) \wedge (S_j \leq S_k) \Rightarrow S_i \leq S_k \tag{4c}$$

Each level S_i consists of a set of objects, $O_0^i \ldots O_n^i$, among which hold the binary topological relations, $t_0 \ldots t_m$. Corresponding objects at different levels will be denoted by a common object index, i.e., O_x^i and O_x^j refer to the same object O_x at levels i and j, respectively. On the other hand, O_x^i and O_y^j will denote two different objects at two different levels. $T(O_x^i)$ will denote the set of all topological invariants for object O_x^i.

Definition 1a: Two representations, $S_i \leq S_j$, are *object-similar* if, for all $O_x^i \in S_i$, $O_x^j \in S_j$:

$$T(O_x^i) \leq T(O_x^j) \tag{5a}$$

where $\leq$ denotes topological object similarity.

Definition 1b: Two representations, $S_i \leq S_j$, are *relation-similar* if, for all $O_x^i, O_y^i \in S_i$, $O_x^j, O_y^j \in S_j$:

$$t(O_x^i, O_y^i) \leq t(O_x^j, O_y^j) \tag{5b}$$

where $\leq$ denotes topological relation similarity.

Definition 1c: Two representations, $S_i \leq S_j$, are *similar* if, for all $O_x^i, O_y^i \in S_i$, $O_x^j, O_y^j \in S_j$, they are object-similar and relation-similar, i.e.,

$$T(O_x^i) \leq T(O_x^j) \quad \wedge \quad T(O_y^i) \leq T(O_y^j) \quad \wedge \quad t(O_x^i, O_y^i) \leq t(O_x^j, O_y^j) \tag{5c}$$

A special case of similarity is the homeomorphism:

Definition 2a: Two representations, $S_i \leq S_j$, are *object-homeomorphic* if, for all $O_x^i \in S_i$, $O_x^j \in S_j$:

$$T(O_x^i) = T(O_x^j) \tag{6a}$$

i.e., if the topology of all pairs of corresponding objects O_x^i and O_x^j is preserved.

Example 1: In Figure 3, the topology of all corresponding objects at Levels 0 and 1 is the same; therefore, the generalization from S_0 to S_1 is object-homeomorphic. Note that the topological relations among some of the objects changed.

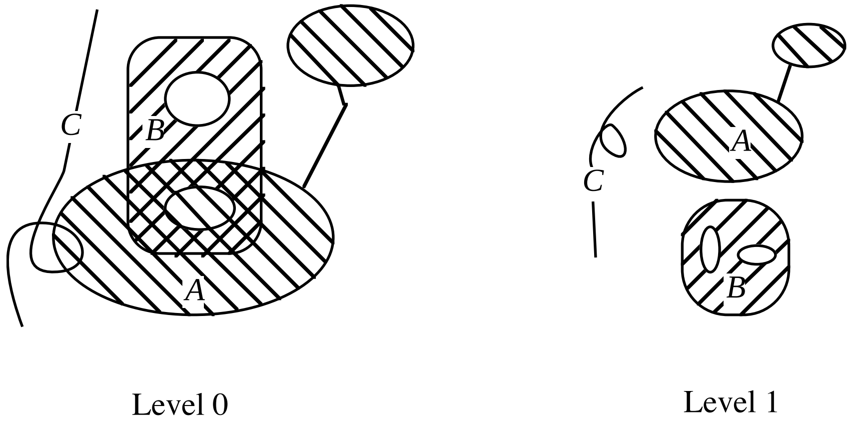


Figure 3: Two object-similar representations.

Definition 2b: Two representations, $S_i \le S_j$, are *relation-homeomorphic* if, for all $O_x^i, O_y^i \in S_i$, $O_x^j, O_y^j \in S_j$:

$$t(O_x^i, O_y^i) = t(O_x^j, O_y^j) \tag{6b}$$

i.e., the topological relation between O_x^i and O_y^i is the same as the one between O_x^j and O_y^j.

Example 2: In Figure 4, the topological relations between the three objects have been retained in the transformation from Level 0 to Level 1, therefore, the generalization is relation-homeomorphic. Whereas the topology of some of the objects changed, none of these changes affected the topological relations among the objects.

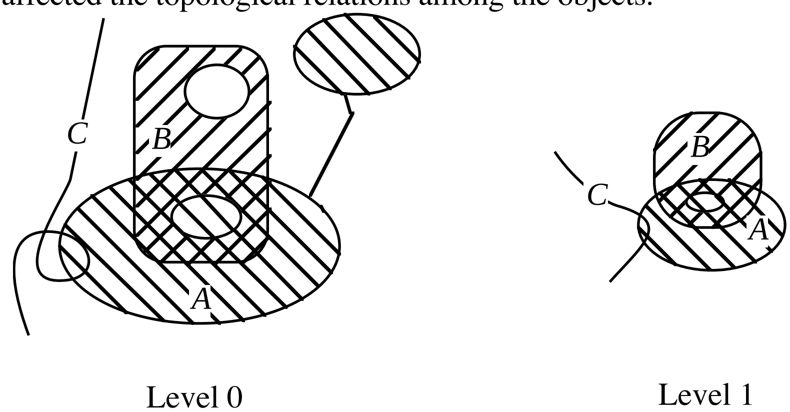


Figure 4: Two relation-similar representations.

Definition 2c: Two representations, $S_i \le S_j$, are *homeomorphic* if, for all $O_x^i, O_y^i \in S_i$, $O_x^j, O_y^j \in S_j$, they are object-homeomorphic and relation-homeomorphic, i.e.,

$$T(O_x^i) = T(O_x^j) \quad \wedge \quad T(O_y^i) = T(O_y^j) \quad \wedge \quad t(O_x^i, O_y^i) = t(O_x^j, O_y^j) \tag{6c}$$

The strongest condition for assessing consistency is when the generalization creates a homeomorphic configuration. While an object-homeomorphic generalization assures that the topology of the individual objects is preserved, it does not guarantee that the topological relation between the objects is preserved. On the other hand, a relation-homeomorphic generalization assures that the dominant topological aspects of the scene are preserved—the topological relations between the objects—while it allows for relatively small changes of individual objects.

4. Object Similarity

Similarity of objects will be based on purely topological measures and no dissimilarities due only to metric properties will be introduced.

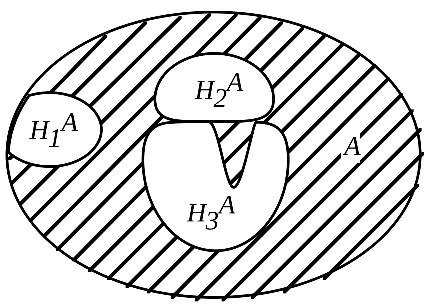
4.1 Object Homeomorphism

Any two homogeneously n-dimensional cells with connected interiors are object-homeomorphic. For instance, a 2-disk is object-homeomorphic to another 2-disk; a non-intersecting line with one start- and one end-node is object-homeomorphic to another non-intersecting line with one start- and one end-node; and a node is object-homeomorphic to another node. Any mapping that does not preserve the topological structure of an object is a deviation from object homeomorphism. For example, a mapping of a 2-disk onto a node is a change in the topology of the object.

The assessment of object homeomorphism becomes more difficult if the objects are more complexly structured. Such an object may include holes, consist of several disconnected parts, or be aggregates of parts of different dimensions. For example, if a 2-dimensional object may include holes, the assessment of object homeomorphism must consider that entire holes may be eliminated or several holes may be aggregated into a single hole. Different levels of similarity may be achieved by eliminating more or less holes, or by different groupings of the holes.

The subsequent discussions focus on the computational assessment of object homeomorphism for regions with holes. For regions with holes, the representation based on generalized regions has been proposed (Egenhofer *et al.* 1994). A *generalized region* is the union of the object and its holes. A region with *n* holes is represented by its generalized region, denoted by A^*, and *n* regions representing the holes, $H_1^A \ldots H_n^A$; each binary topological relations between A^* and $H_1^A \ldots H_n^A$, and the set of all binary topological relations among $H_1^A \ldots H_n^A$. This set of relations can be represented concisely in the form of a matrix, called the *relation matrix R*. Within a relation matrix, a number of constraints must hold among the individual entries. For example, each object must be *equal* to itself and the relation between two objects A and B must be converse to the relation between B and A (Egenhofer and Sharma 1992). Additional constraints exist depending on whether the objects represent the generalized region or the holes: the topological relations between a generalized region and its holes must be *contains* and *covers*, while the topological relations between holes must be *disjoint* or *meet.*

In order to establish object homeomorphism for regions with holes, the representation of generalized regions must be extended. First it is necessary to distinguish between two holes that *touch* along an edge or in a point (Egenhofer and Herring 1990). There may be refinements due to the dimension of the boundary-boundary intersection for *covers* as well. Second, holes may have more than one common boundary part with another hole, called a *boundary-boundary component*, each of which may have different dimensions. Third, combinations of components that *meet* (or *cover*) in different dimensions may occur in different sequences (Franzosa and Egenhofer 1992). Therefore, one has to compare (1) the topological relations between the generalized region and its holes and (2) for any *meet* or *covers* relation, the sequence of the components and their dimensions. Figure 5 shows a region with holes, its relation matrix, and the component invariant tables for non-empty boundary-boundary intersections.



	A^*	H_1^A	H_2^A	H_3^A
A^*	equal	covers	contains	contains
H_1^A	coveredBy	equal	disjoint	disjoint
H_2^A	inside	disjoint	equal	meet
H_3^A	inside	disjoint	meet	equal

$$T(\partial H_2^A, \partial H_3^A) = \begin{bmatrix} 1 & 2 \\ \hline 1 & 0 \end{bmatrix}$$

$$T(\partial A^*, \partial H_1^A) = \begin{bmatrix} 1 \\ \hline 1 \end{bmatrix}$$

Figure 5: (a) A region *A* with three holes $H_1^A \ldots H_3^A$ and (b) the relation matrix for the generalized region, A^* and the component invariant tables for the boundary-boundary intersections between H_2^A and H_3^A, and between A^* and H_1^A.

4.2 Strategies for Establishing Object Similarity

Different degrees of object similarity are described by more or less deviations from object homeomorphism. They can be formally expressed as changes in the component invariants. In order to comply with the monotonicity assumption, which introduces an order between representations, it is necessary that the topological invariants that change from one representation to another, are at least on an ordinal scale of Steven's (1946) categorization of measurements. For nominal properties, a meaningful generalization is impossible.

Among the component invariants that are candidates for changes, there are four that fulfill this constraint: dimension (0, 1, …), number of boundary-boundary components (0, 1, …), and number of holes (0, 1, …) are on a ratio scale and, therefore, also ordinal. The topological relations (between two holes or between the generalized region and a hole), are also candidates for changes because they are (partially) ordered through the conceptual neighborhood (Egenhofer and Al-Taha 1992). The only invariant that is on a nominal scale is the sequence of boundary-boundary components, because two different sequences of the same number of components cannot be compared for order.

The question remains into which *direction* the values of the invariants should change: Does the number of boundary-boundary components grow or shrink? Likewise, should the dimension of a boundary-boundary component between two holes increase or decrease? Since the overall goal is to reduce the complexity of an object, the number of significant parts should become smaller. For objects with holes, this means that the number of holes will be reduced, either by aggregating two or more holes into a single hole, or by dropping a less significant hole. This guides any decision about the direction into which the invariants should change. The *number of holes* should *decrease* towards 0, otherwise more additional complexity would be introduced. Likewise, the *number of boundary-boundary components* between two holes or between the generalized region and a hole should get *smaller*. If the *topological relation* between two holes changes then it must be a change *from disjoint to meet*, otherwise more distinguishable holes would be introduced. This means also that the *dimension* of a boundary-boundary component between two holes, or between a hole and the generalized region, may *increase* from one representation level S_i to another representation level S_{i+1}. For example, a component intersection that *meets* in a node between H_1^A and H_2^A may change to a *meet* in an edge between S_i and S_{i+1} when the two holes are moved closer to each other.

4.3 Assessing Object Similarity

The easiest assessment is the change in the dimension of one or several relations, because the relation matrix must be preserved and only the dimension changes in the corresponding component invariant table. More difficult is the comparison if holes are dropped or aggregated as this not only eliminates this hole's relations with respect to the other holes, but may also imply changes in the other topological relations. To aggregate two holes into one, the two holes must have at least one boundary-boundary component that *meets* in an edge.

- If H_i^A is connected to another hole H_x^A through a relation t_i and H_j^A is *disjoint* from H_x^A, then t_i immediately applies to the relation between $H_{i\otimes j}^A$ and H_x^A.
- If H_i^A and H_j^A are linked to the same hole, H_x^A, through the relations $t_i \neq$ *disjoint* and $t_j \neq$ *disjoint*, and there is a component of H_x^A's boundary that lies between the components of t_i and t_j and its relation is $t_k \neq$ *disjoint*, then t_i and t_j apply both to the relation between $H_{i\otimes j}^A$ and H_x^A, otherwise, t_i and t_j are integrated into a single relation.

The relations of a configuration with aggregated holes are determined according to the following rules (the pattern is: $t(H_i^A, H_x^A) \otimes t(H_j^A, H_x^A) \rightarrow t(H_{i\otimes j}^A, H_x^A)$, where H_i^A and H_j^A are the two holes to be aggregated, $H_{i\otimes j}^A$ is the aggregated hole, and the aggregation operation $\otimes$ is commutative):

$$t_i \otimes \textit{disjoint} \rightarrow t_i \quad (7a)$$

$$\textit{meet} \otimes \textit{meet} \rightarrow \textit{meet} \quad (7b)$$

For relations between the holes and the generalized region:

$$t_i \otimes inside \rightarrow t_i \tag{8a}$$

$$coveredBy \otimes coveredBy \rightarrow coveredBy \tag{8b}$$

For the dimensions between the holes and the generalized region:

$$n\text{-dimensional} \otimes m\text{-dimensional} \rightarrow \max(n, m)\text{-dimensional} \tag{9}$$

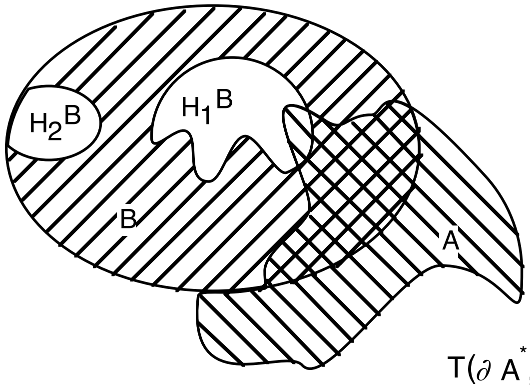
The same principles apply when dropping a hole.

5. Relation Similarity

5.1 Relation Homeomorphism

An assessment of relation homeomorphism between regions must consider all component invariants of the 4-intersection, i.e., the content, the sequence of the components, their dimensions, their types (*touching*, *crossing*, and different refinements of *crossings*), and their relationships with respect to the exterior neighborhoods (Egenhofer and Franzosa 1993); therefore, two binary topological relations between simple regions *A* and *B,* and *C* and *D,* respectively, are relation-homeomorphic if they have the same 4-intersections, i.e., $I(A, B) = I(C, D)$; *and* for $\partial A \cap \partial B = \neg\emptyset$ and $\partial C \cap \partial D = \neg\emptyset$, the same component invariants, i.e., $T(\partial A, \partial B) = T(\partial C, \partial D)$.

The assessment of relation homeomorphism for regions with holes is more complex, because it is necessary to guarantee that all relations in which holes are involved, are preserved. Holes that are not involved are those that are *disjoint* from the other object's generalized region. Such holes do not influence the topological relation between the complex objects and, therefore, may be dropped as one generalizes from one level to a less detailed level. All other relations among the generalized regions and the holes must remain the same.



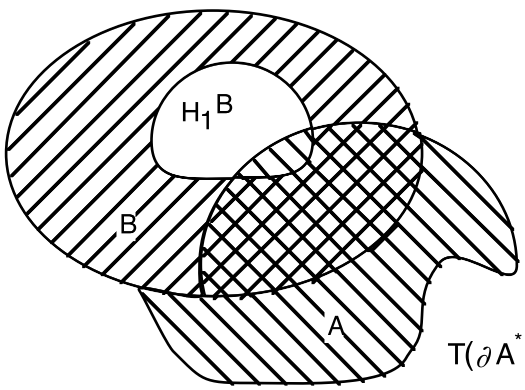
(a)

	B^*	H_1^B	H_2^B
A^*	overlap	overlap	disjoint

(c)

$$T(\partial A^*, \partial B^*) = \begin{bmatrix} 2 & 1 \\ \hline 1 & 0 \\ icl & cO \\ u & u \end{bmatrix} \quad T(\partial A^*, \partial H_1^B) = \begin{bmatrix} 2 & 1 \\ \hline 0 & 0 \\ cl & cO \\ u & u \end{bmatrix}$$

(e)



(b)

	B^*	H_1^B
A^*	overlap	overlap

(d)

$$T(\partial A^*, \partial B^*) = \begin{bmatrix} 2 & 1 \\ \hline 1 & 0 \\ icl & cO \\ u & u \end{bmatrix} \quad T(\partial A^*, \partial H_1^B) = \begin{bmatrix} 2 & 1 \\ \hline 0 & 0 \\ cl & cO \\ u & u \end{bmatrix}$$

(f)

Figure 6: The two configurations (a) and (b) are relation-homeomorphic, because they have the same relation matrices (c) and (d) except for the fact a column with only *disjoint* relations has been dropped; and identical component invariant tables (e) and (f) for the relations with non-empty boundary intersections.

The notation $M(A, B) \nabla M(C, D)$ is introduced to denote that two relation matrices are the same, except that rows and columns of holes that are *disjoint* from the other object's generalized region may have been dropped. This gives rise to the definition of relation homeomorphism: Two binary topological relations between two pairs of regions with holes are homeomorphic if $M(A, B) \nabla M(C, D)$ and if they have the same component tables for non-empty boundary-boundary intersections, i.e., $T(\partial a, \partial b) = T(\partial c, \partial d)$, where $a \in \{A^*, H_1^A, \ldots, H_n^A\}$, etc. Figure 6 shows an example of two relation-homeomorphic configurations and the formalism how relation homeomorphism was determined.

5.2 Strategies for Establishing Relation Similarity

Different degrees of relation similarity can be obtained by allowing certain changes in the invariants. Some of the invariants are fairly complex and any change in them becomes a serious change in the topological relation that would violate the monotonicity assumption. Invariants that are on a nominal scale and, therefore, cannot be changed are the sequence of boundary-boundary components; the type of a component intersection (e.g., from *inner cross* to *outer cross* or vice versa; from *crossing into* to *crossing out of*, or vice versa; and from *touch* to a *cross* or vice versa), and the boundedness of a component.

The remaining invariants are at least on an ordinal scale:

- the dimension of a component;
- the number of boundary-boundary components; and
- the number of holes.

5.3 Assessing Relation Similarity

Provided the two regions keep the same number of holes and component intersections, the only change that will retain a relation-similar situations is a decrease of the dimension of *crossing* or *touching* components from 1 to 0. While this change happens independent of most other component invariants, for *crossing* components it relates immediately to the crossing type, i.e., *inner/outer crossing*. Since *inner/outer crossing* is a property associated only with 1-dimensional *crossing* components, the transition from a 1-dimensional *crossing* to a 0-dimensional *crossing* has also to cancel the inner/outer information. In the formalism, changing one component's dimension invariant affects the corresponding column of the component invariant table. Equation 10 shows an example of the relevant parts of the two component invariant tables.

$$\begin{pmatrix} 1 \\ \hline 1 \\ \mathrm{icO} \\ \mathrm{u} \end{pmatrix} \xrightarrow{\leq} \begin{pmatrix} 1 \\ \hline 0 \\ \mathrm{cO} \\ \mathrm{u} \end{pmatrix} \qquad (10)$$

More types of changes are possible if the number of holes or the number of component intersections changes. Following the monotonicity assumption, both numbers cannot increase because otherwise more detailed, rather than more general configurations would result. In our formalism, such changes are expressed as a reduction of the number of columns in the component invariant table. While such eliminations are straightforward, it is necessary to maintain consistency among the remaining elements in the component invariant table. For example, when dropping a *touching* component, the corresponding column is eliminated and the boundedness of the adjacent components has to be checked: if the *touching* component is *unbounded* and there exists an adjacent *bounded* component, then that component must become *unbounded*. Equation 11 shows an example of the relevant parts of the two component invariant tables.

$$\begin{pmatrix} \underline{1 \quad 2} \\ 0 \quad 0 \\ cO \quad t \\ b \quad u \end{pmatrix} \xrightarrow{\leq} \begin{pmatrix} \underline{1} \\ 0 \\ cO \\ u \end{pmatrix} \tag{11}$$

The same principle applies when eliminating *crossing* components, however, unlike *touching* components, they must be eliminated in pairs of consecutive components. The consistency constraint with respect to the remaining columns is that if one of the two *crossing* components to be eliminated is unbounded and the other component is adjacent to a bounded component, then that component will turn into an unbounded component (Equation 12).

$$\begin{pmatrix} \underline{3 \quad 2 \quad 1} \\ 0 \quad 0 \quad 0 \\ cI \quad cO \quad t \\ u \quad b \quad b \end{pmatrix} \xrightarrow{\leq} \begin{pmatrix} \underline{1} \\ 0 \\ t \\ u \end{pmatrix} \tag{12}$$

In a similar way, the aggregation of boundary-boundary components can be formalized. Again, the components must be consecutive, otherwise, they cannot be aggregated. While the focus will be on the aggregation of two components, aggregates of more than two components can be formed by recursively applying the same method to the result and the next component.

Based on the types of component intersection (*crossing* and *touching*), there are three combinations of interest:

- Two *crossing* components can be aggregated into a 1-dimensional *touching* component. The *touching* component will be *unbounded* if one of the two *crosses* was *unbounded*, otherwise it is *bounded* (Equation 13).

$$\begin{pmatrix} \underline{2 \quad 1} \\ 0 \quad 1 \\ cI \quad icO \\ b \quad u \end{pmatrix} \xrightarrow{\leq} \begin{pmatrix} \underline{1 \otimes 2} \\ 1 \\ t \\ u \end{pmatrix} \tag{13}$$

- A *crossing* and a *touching* component can be aggregated into a 1-dimensional *crossing* component. Such a *crossing* component will be *unbounded* if at least one of the two initial components was *unbounded*, otherwise it is *bounded*. Information about the direction (*into* or *out of*) of the new *crossing* component propagates directly from the initial *crossing* component. Finally, *outer crossing* is inferred from an ascending sequence of the components intersections, whereas *inner crossing* follows from a descending sequence (Equation 14).

$$\begin{pmatrix} \underline{4 \quad 3} \\ 0 \quad 0 \\ cO \quad t \\ b \quad b \end{pmatrix} \xrightarrow{\leq} \begin{pmatrix} \underline{3 \otimes 4} \\ 1 \\ icO \\ b \end{pmatrix} \tag{14}$$

- Two *touching* components can be aggregated into a 1-dimensional *touching* component. Such a *touching* component will be *unbounded* if one of the two initial component intersections was *unbounded*, otherwise it is *bounded* (Equation 15).

$$\begin{pmatrix} \underline{2 \quad 1} \\ 1 \quad 0 \\ t \quad t \\ b \quad u \end{pmatrix} \xrightarrow{\leq} \begin{pmatrix} \underline{1 \otimes 2} \\ 1 \\ t \\ u \end{pmatrix} \tag{15}$$

6. Conclusions

This paper presented a framework within which topological inconsistencies across multiple representations can be evaluated. It applies to multiple-representation GISs, as well as to some stages of automated map generalization as a method for checking topological consistency. Finally, the framework is useful for data fusion when geographic objects and their spatial relations, have to be compared and integrated. The assessment of consistency is governed by the monotonicity assumption, under which changes through a series of different representations must be orderable. We developed computational models to evaluate the consistency of a single object, as well as the consistency between objects. These models focused on such properties as holes in regions or the topological relations between regions.

This theory provides a first step towards the computational assessment of multiple representations. More work is necessary to formalize the changes if objects are (1) split into separations, (2) modeled as lines or points, and (3) heterogeneously composed of areal and linear parts. Besides the development of the theory, it will be useful to calibrate the model with existing multiple-representation data sets, such as digital map series over multiple scales, from which more insight will be gained about how human cartographers make decisions in such complex processes as cartographic generalization.

7. References

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