Approximations of Geospatial Lifelines
(Extended Abstract)

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1. Introduction

A *geospatial lifeline* models continuous movement through geographic space as a time-stamped record of locations that an object has occupied over a period of time (Hornsby and Egenhofer 2002). Unlike traditional models of moving points that fill the gaps between the recorded space-time samples through interpolation to determine a *likely* space-time point, a *lifeline bead* captures the set of *all possible* space-time points that the object may have occupied between the space-time samples based on heuristics about the object's maximum travel speed. Such a lifeline bead is modeled as the intersection of two halfcones pointing in opposite directions—one from the origin pointing in the direction of the time axis, and one from the destination pointing in opposite direction. The aggregate of simply connected beads forms then a *lifeline necklace*. Such lifeline necklaces provide the bases for analyses that are complementary to traditional moving-object queries. For example, two individuals' records of their time-stamped locations may be used to dismiss an alibi if the records show that it was impossible that the two suspected individuals could have met. Likewise, the analysis of lifeline necklaces may provide insights as to whether an individual might have been at a particular location during a particular time window.

2. Lifeline Beads

The foundation for performing these kinds of analyses is the calculation of the intersection of two lifeline beads. The model of a *lifeline bead* is based on the geometry of a *circular right cone* (i.e., a cone with a vertical rotation axis and the directrix takes the shape of a circle that is located in a plane orthogonal to the cone's rotation axis). The directrix radius r, the cone's height h as the difference between the z-values of the apex and the directrix, , and the apex angle ϕ as the surface's slope with respect to the vertical rotation axis determine a right circular cone (Equation 1a-c).

$$x = \frac{h-t}{h} r \cos\phi \tag{1a}$$

$$y = \frac{h-t}{h} r \sin\phi \tag{1b}$$

$$z = t \tag{1c}$$

The set of all possible movements an individual might have taken between two consecutive sample observations is modeled as the intersection of two halfcones whose directrices face back to back—the lower halfcone for all possible space-time points visited from a specified *origin* (x_0, y_0, t_0) (Equation 2a), and the upper half cone for the space-time points at which the individual could have been while approaching the *destination* (x_1, y_1, t_1) (Equation 2b), with $t_0 \le t \le t_1$ and the apex angle ϕ (i.e., the surface's slope with respect to the cone's vertical rotation axis). The intersection of the two halfcones is referred to as a *lifeline bead*.

$$(x - x_0)^2 + (y - y_0)^2 \le (t - t_0)^2 \tan^2 \phi \quad (2a)$$

$$(x - x_1)^2 + (y - y_1)^2 \le (t - t_0)^2 \tan^2 \phi \quad (2b)$$

If the origin (x_0, y_0, t_0) and the destination (x_0, y_0, t_1) are at the same location, then the apexes of a bead are collocated in space, but shifted in time, forming a *right bead* (Figure 1a). Since the two apex angles from opposite sides are identical, the intersection between the two halfcones forms a circle, located in the plane $t = \frac{(t_0 + t_1)}{2}$, around the center $\left(x_0, y_0, \frac{(t_0 + t_1)}{2}\right)$, and with a radius $r = \frac{t_1 - t_0}{2} \tan \phi$.

On the other hand, if a bead's origin (x_0, y_0, t_0) and destination (x_1, y_1, t_1) are at different locations (i.e., $x_0 \ne x_1$ or $y_0 \ne y_1$), then the apexes of the bead are not spatially aligned, forming an *oblique bead* (Figure 1b). The projection of an oblique bead into the plane $t = t_n$ results in an ellipse whose foci are the projections of the origin (x_0, y_0, t_n) and destination (x_1, y_1, t_n), and its major axis ($a = \frac{1}{2} \Delta t_{01} \tan \phi$) and minor axis ($b = \sqrt{a^2 - \frac{d_{01}^2}{4}}$) depend on the apex angle ϕ, the time difference Δt_{01}, and the planar distance d_{01} from the origin to the destination. For a right bead, the projections of the origin and the destination coincide around which the projected figure forms a circle with the same radius as the intersection of the two cones ($r = \frac{\Delta t_{10} \tan \phi}{2}$).

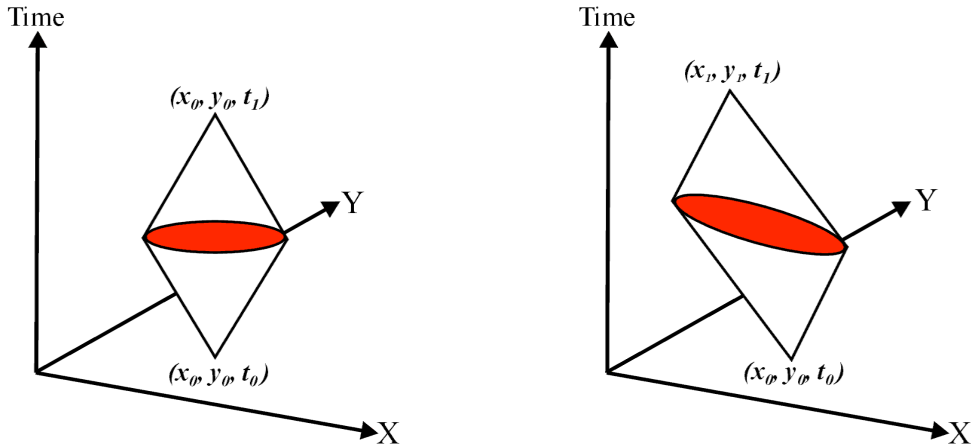

Figure 1: Lifeline beads: (a) a right bead and (b) an oblique bead.

In general, calculations for testing intersections on such lifeline beads rely on the intersections of four quadrixes, which are operations of high complexity. Although a set of configurations for which less expensive intersection tests apply has been derived (Hariharan 2001), additional methods need to be employed to provide performance improvements of lifeline intersection tests.

3. Approximations of Lifeline Beads: Minimum-Bounding Boxes

The performance of intersection tests can be improved at times by considering first a coarser approximation of beads, on which intersections can be determined more simply than on their actual geometry. Approximations are typically used in spatial databases to provide a fast filter for spatial queries, such that only a small subset of candidates needs to be examined through more detailed methods rather than applying the elaborate methods for every operation (Kriegel *et al.* 1991). Each approximation fully encloses the corresponding object, therefore, if two objects intersect, then their approximations must intersect as well. The reverse, however, it not always true, since a positive intersection test on the approximations is not necessarily conclusive for the intersection of the actual shapes.

The choice of the best geometric approximation typically depends on heuristics about the shape of the actual objects. Most common in geographic applications is the use of *minimum-bounding rectangles* (MBRs) as approximations of arbitrary two-dimensional objects. Since beads are 3-dimensional objects, MBRs need to be generalized from 2-dimensional rectangles (with their sides parallel to the two coordinate axes) to 3-dimensional boxes (MBBs) with their three sides parallel to the three orthogonal coordinate axes. To describe completely an MBB, one needs the three-dimensional coordinates of two opposite corners. The highest ($z_{MBB}^{\min}(A) = \Omega_l(A)$) and lowest ($z_{MBB}^{\max}(A) = \Omega_u(A)$) points of a bead's MBB correspond to its height, yielding two of the six values. The MBB's extent in the XY plane forms an MBR whose opposite corners provide the remaining four values. This MBR must enclose the bead's projection into the XY plane—either a circle for a right bead or an ellipse for an oblique bead. Since the ellipse's axes are parallel to the MBB's sides only if $\Delta x_{10} = 0$ or $\Delta y_{10} = 0$, the bounding box needs to be derived from calculating the ellipse's horizontal and vertical tangents. The major axis *a* and the minor axis *b* are given by the bead's projection into the XY plane.

4. Approximations of Lifeline Beads: Minimum-Bounding Circular Cylinders

The particular shape of right and oblique beads may favor other geometric shapes that offer better approximations. A right circular cylinder that has a base that is parallel to the x-y plan and tightly encloses a bead is referred to as a *minimum-bounding circular cylinder* (MBCC). Like the MBB's height, the height of an MBCC coincides with the interval formed by the bead's apexes (i.e., $z_{MBCC}^{\min}(A) = \Omega_l(A)$ and $z_{MBCC}^{\max}(A) = \Omega_u(A)$). For a right bead, the MBCC's circular base coincides with the bead's directrix such that $x_{MBCC}^{center} = x_0$, $y_{MBCC}^{center} = y_0$, and $r_{MBCC} = \Delta t_{10} \tan \phi$. For an oblique bead, the MBCC's base is a circle whose center is the middle between the departure and arrival points ($x_{MBCC}^{center} = \frac{x_0 - x_1}{2}, y_{MBCC}^{center} = \frac{y_0 - y_1}{2}$), while the MBCC's radius corresponds to the major axis of the projected bead ($r_{MBCC} = \frac{1}{2} \Delta t_{10} \tan \phi$). The conclusion from MBCC tests is that two beads do not intersect if their corresponding MBCCs do not intersect. This constraint requires that the two MBCCs are disjoint along the z-axis ($z_{MBCC}^{\max}(A) < z_{MBCC}^{\min}(B) \lor z_{MBCC}^{\min}(A) > z_{MBCC}^{\max}(B)$) and that the distance between the centers of the two footprints is greater than the sum of the two MBCCs' radii ($(x_{MBCC}^{center}(A) - x_{MBCC}^{center}(B))^2 + (y_{MBCC}^{center}(A) - y_{MBCC}^{center}(B))^2 > (r_{MBCC}(A) + r_{MBCC}(B))^2$). The intersection constraints in the x-y plane projection correspond to those of two circles in the plane (Safar and Shahabi, 1999).

5. Approximations of Lifeline Beads: Minimum-Bounding Elliptic Cylinders

An even better approximation for a bead is a *minimum-bounding circular cylinder* (MBEC), that is, a right cylinder with an ellipse as its base. This ellipse is formed by the projection of the intersection of the upper and lower halfcones into the XY plane. While MBECs are best approximations, operations for testing MBEC intersections are significantly more complex than MBCC intersections.

6. Summary

Sequences of lifeline beads support the conclusion of a *potential meeting* between two individuals that move through space, or dismissal of a hypothesis that they could have met. The computational foundation for this analysis relies on intersections of four half-cones, an expensive operation. Similar to typical spatial search operations, approximations of lifeline beads offer a fast filter for the intersection operation. However, unlike the ubiquitous minimum-bounding rectangles and their generalization to minimum-bounding boxes in 3-dimensional space, better approximations for the lifeline-bead intersection test are minimum-bounding circular cylinders or minimum-bounding elliptic cylinders.

	Closeness of approximation	Complexity of constructing the approximation	Complexity of intersection tests
MBB	medium	high	low
MBCC	medium	low	medium
MBEC	high	low	high

7. References

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