

Robust Inference of the Flow Direction in River Networks*

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Abstract

An algorithm has been designed to infer the flow direction of a river network represented in a vector data model. It is based on the connectivity of channels and heuristics about the angles at which channels intersect at junctions, but it requires no information about the elevation of the terrain. The algorithm finds first the main branches of a network, from which it then infers the destination. Empirical tests with digitized river networks have demonstrated the robustness of the algorithm.

1 Introduction

River or drainage networks are fundamental concepts used for various analyses in the geosciences. Geologists, for instance, derive original slope and original structure from drainage patterns. Likewise, transportation engineers examine river networks to determine how to access undeveloped land via waterways. Frequently, information about river networks only consists of the connectivity of channels, lacking any explicit information about the flow direction of the network. Examples of such information sources are remotely sensed images, aerial photographs, and maps without explicit elevation information such as contour lines. Despite the absence of explicit information about the flow directions, humans perform very well in identifying the flow direction of a river network from a planimetric representation (Figure 1). The formalization of such reasoning processes, however, has proven to be very difficult and no reliable and robust methods exist for implementations on a computer. Since many analysis methods for drainage networks assume that the flow direction is known [2, 17], it is necessary to automatically detect the outlet(s) of a river network.

* This work has been partially supported by the Brazilian National Research Council (CNPq) under grant number 260226/91.2 and the National Science Foundation for the NCGIA under grant No. SES 88-10917. João Paiva's work is currently supported by CNPq under grant number 200073/94-0, and Max Egenhofer's research has been supported by grants from the National Science Foundation under grant number IRI-9309230; Rome Laboratories under grant number F30602-95-1-0042; the Scientific and Environmental Division of the North Atlantic Treaty Organization; Intergraph Corporation; Environmental Systems Research Institute Inc.; and by a Massive Digital Data Systems contract sponsored by the Advanced Research and Development Committee of the Community Management Staff and administered by the Office of Research and Development.

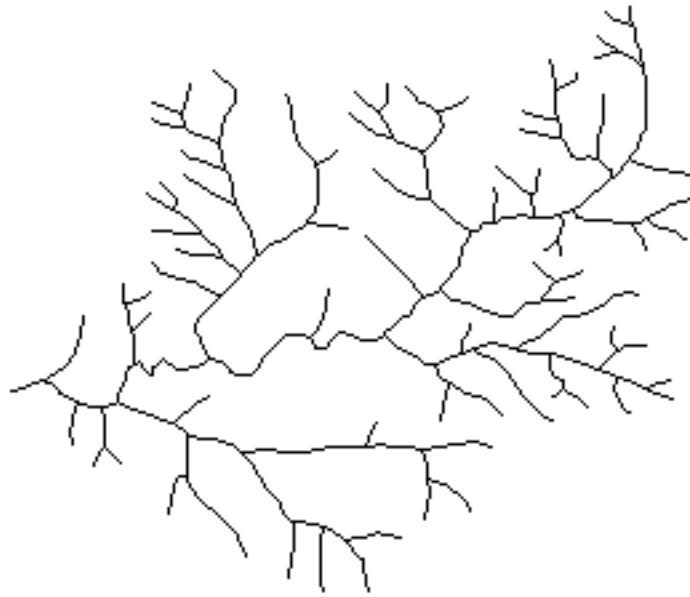


Figure 1: A planimetric representation of a river network, for which the human eye easily detects the likely outlet.

This paper presents a new computational method to find the location of the destination of a river network and, therefore, infer its flow direction. The approach taken here differs considerably from earlier attempts to reason about flow directions as it exploits expert knowledge about geological processes and integrates it into our algorithm. Usually, additional information from a digital elevation model is required to complete the inference of the flow directions [1, 5, 6, 13, 16]. While such an approach may be appropriate for steep terrain with significant elevation differences, it is unfeasible in flat terrain. *In lieu* of using elevation information, we base our computational method exclusively on information about the connectivity of the network channels and the angles at which channels meet at junctions. The geological expert knowledge is founded upon the early recognition that the angles at which stream segments join contain crucial information for the inference of the flow directions in river networks [10, 15]. Investigations of river networks revealed that the most acute angle encloses the upstream channels for over 80% of all junctions [3]. We exploit this knowledge together with the additional knowledge that the main branch is usually formed by the two channels whose junction angle is closest to 180° . Both heuristics about the junction angles represent common geological knowledge about the processes that form river networks.

In previous work, flow directions have been inferred by skeletonizing the water channels, obtained from remotely sensed images, and by applying a set of constraint rules about the junctions (angles and channel lengths) of river channels [8, 18]. The algorithms used to propagate information about the flow direction within a network deal with incomplete information, because the flow directions of some channels are unknown; however, they assume that all known flow directions are correct. This is different from our approach in which we allow redundant and sometimes contradicting information about flow directions.

The remainder of this paper is structured as follows: In section 2 we introduce a model for river networks based on graph theory. Subsequently, we present an algorithm to infer the flow direction of a river network from the topology of channels and the angles at which channels meet at

junctions (Section 3) and describe the results from testing the algorithm with several digitized river networks (Section 4). Our conclusions are presented in section 5.

2 Modeling River Networks

The formal basis for modeling river networks is the mathematical structure of a *graph*. The following fundamental definitions are based on Knuth [11] and Gill [7]. A *directed graph* is a set of vertices and a set of arcs such that each arc leads from an initial vertex V to a final vertex V' . The *orientation* of an arc is an equivalence class, defined as a positive value *from* the initial vertex *to* the final vertex, and negative in the reverse direction. The *out-degree* of a vertex V is the number of arcs leading out from it (i.e., the number of arcs e , whose initial vertex is V). Conversely, the *in-degree* of V is the number of arcs whose final vertex is V . The *degree* of a vertex V is then the sum of the in-degree and out-degree of V . A directed graph is depicted as a sequence of nodes (for the vertices) and edges between the nodes (for the arcs). The orientation of each arc is represented by an arrow pointing from the node for the initial vertex to the node for the final vertex.

The most common river junction patterns can be formalized in terms of graph-theoretic concepts [14]. We use the terms *node* and *channel* respectively for a vertex and an arc in a river network. Nodes are classified based on their in-degrees and out-degrees. A *source* is a node of out-degree 1 and in-degree 0. The *destination* of a network is a node that has in-degree 1 and out-degree 0. *Auxiliary nodes* are vertices whose in-degree and out-degree are 1. Unless such nodes are specific features, such as lakes, they contain no topologically significant information and can be eliminated. A *junction* is a node of out-degree 1 and in-degree 2 or higher. The in-degree of a junction corresponds to the number of its *upstream channels*, while the out-degree is a measure for the number of channels *downstream* from the junction. An *internal channel* is a channel whose nodes are junctions, whereas an *external channel* has at least one node of degree 1. Two channels are *adjacent* if they share a common node, called the *internal channel node*, one of which being an upstream channel and the other a downstream channel.

The *junction geometry* describes the metric information among the related channels. The primary geometric concern is the information about the angles at which the channels flow together. Two heuristics about the junction geometry are of particular importance: (1) the 180° -assumption and (2) the acute-angle assumption.

- The *180° -assumption* anticipates that the predominant continuation of the flow direction is along the major channel. Usually, one of the two upstream channels is stronger and has the major influence in the continuation of the flow. Unless there are serious geological influences, the water flow of the stronger channel tends to continue its flow direction, while the minor channel joins the major channel and changes its flow direction. Therefore, the upstream channel that forms an angle closest to 180° with the downstream channel is considered to be part of the main channel and is called the *parent channel*. The other upstream channels at the same junction are called *tributaries*.
- The *acute-angle assumption* presumes that the two consecutive channels that enclose the most acute angle are the upstream channels. It is based on an inference mechanism about the channel flows that is based solely on angles at river junction. For nine distinct geometries at a junction with three channels, de Serres and Roy found a set of inference rules (Table 1) that match closely with dendritic river networks. Empirical tests demonstrated that this is true for about 88% of the junctions in dendritic networks [3].

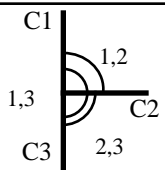
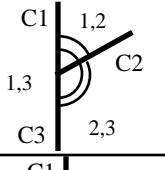
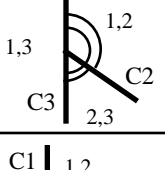
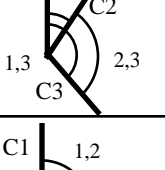
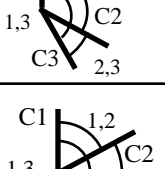
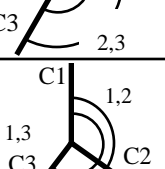
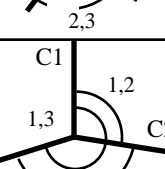
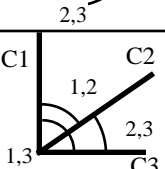
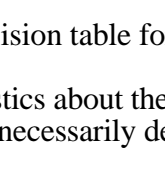
Case	1,3	1,2	2,3	Downstream channel
1 	$= 180^\circ$	$= 90^\circ$	$= 90^\circ$	C1 or C3
2 	$= 180^\circ$	$< 90^\circ$	$> 90^\circ$	C3
3 	$= 180^\circ$	$> 90^\circ$	$< 90^\circ$	C1
4 	$< 180^\circ$	90°	90°	C3
5 	$< 180^\circ$	90°	90°	C1
6 	$> 180^\circ$	90°	90°	C3
7 	$> 180^\circ$	90°	90°	C1
8 	$> 180^\circ$	90°	90°	C3 if $1,2 < 2,3$ C1 if $2,3 < 1,2$
9 	$< 180^\circ$	$< 90^\circ$	$< 90^\circ$	C3 if $1,2 < 2,3$ C1 if $2,3 < 1,2$

Table 1: Decision table for network junctions based solely on angles (from [3]).

These heuristics about the junction geometry are directly related to dendritic drainage patterns, and they do not necessarily describe the angle junction behavior of other drainage patterns.

3 Branch Topology

This section introduces our model for a river network, which is based on branches of channels. A *branch* is an ordered sequence of recursively adjacent channels. This concept has been called a stream in Horton’s [9] stream ordering model. A node is *internal* to a branch if it is also an internal channel node of two of its adjacent channels. Branch nodes that are not internal to a branch are called *external branch nodes*. A branch B_i flows into another branch B_j if an external branch node of B_i coincides with an internal branch node of B_j . Branches are used to represent the connectedness between (non-directed) channels in order to identify the likely outlet channel of a river network. We make two assumptions about the network: (1) all channels are connected and (2) only sources, junctions, and destinations are represented, while any auxiliary nodes, such as non-junction nodes that describe details about the geometry, have been eliminated [14]. The basic information provided is a set of 3-tuples (channelID, nodeID, nodeID), which describe each channel by its start and end node; and 4-tuples (nodeID, channelID, channelID, angle) to capture at each node the clockwise angles between the two channels.

The process of generating branches involves four steps: (1) finding evidence for the flow direction of each channel, (2) determining for each channel its parent channel; (3) identifying the branches of a network, and (4) inferring the branch with the likely outlet. The network in Figure 2 is used as the reference for the examples in this section. Nodes are numbered N1...N20 and channels C1...C19.

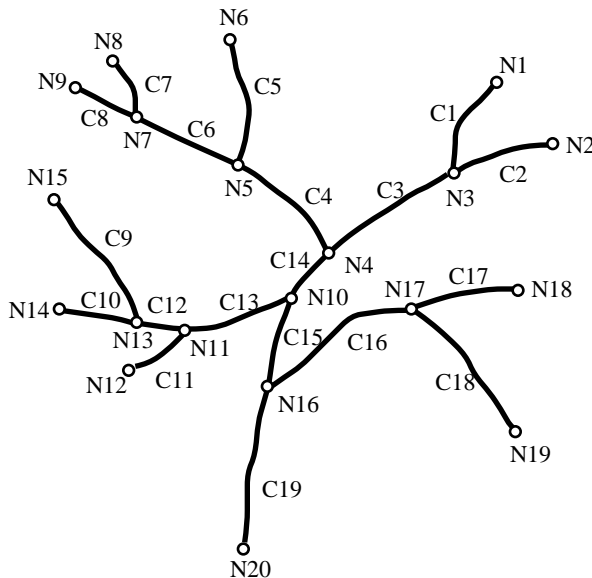


Figure 2: Example of a river network. Nodes and channels are numbers by N* and C*, respectively.

3.1 Estimated Flow Direction

We use the acute-angle assumption to gain evidence for the flow direction. This process assigns to each internal channel two flow directions, and one flow direction to each external channel (Figure 4). The flow direction is undefined if two junction angles are exactly 90° . The outcome of this first step is a set of 3-tuples (channelID, nodeID, flowDirection), describing the flow direction—upstream or downstream—of each channel with respect to the start or end node. The combined results from the angle rules may be sometimes inconsistent, because for the same interior

channel, two contradicting flow directions may have been determined (e.g., Channels C14 and C15 in Figure 3). At this point, we are not concerned with such inconsistencies as they will be resolved in the upcoming steps.

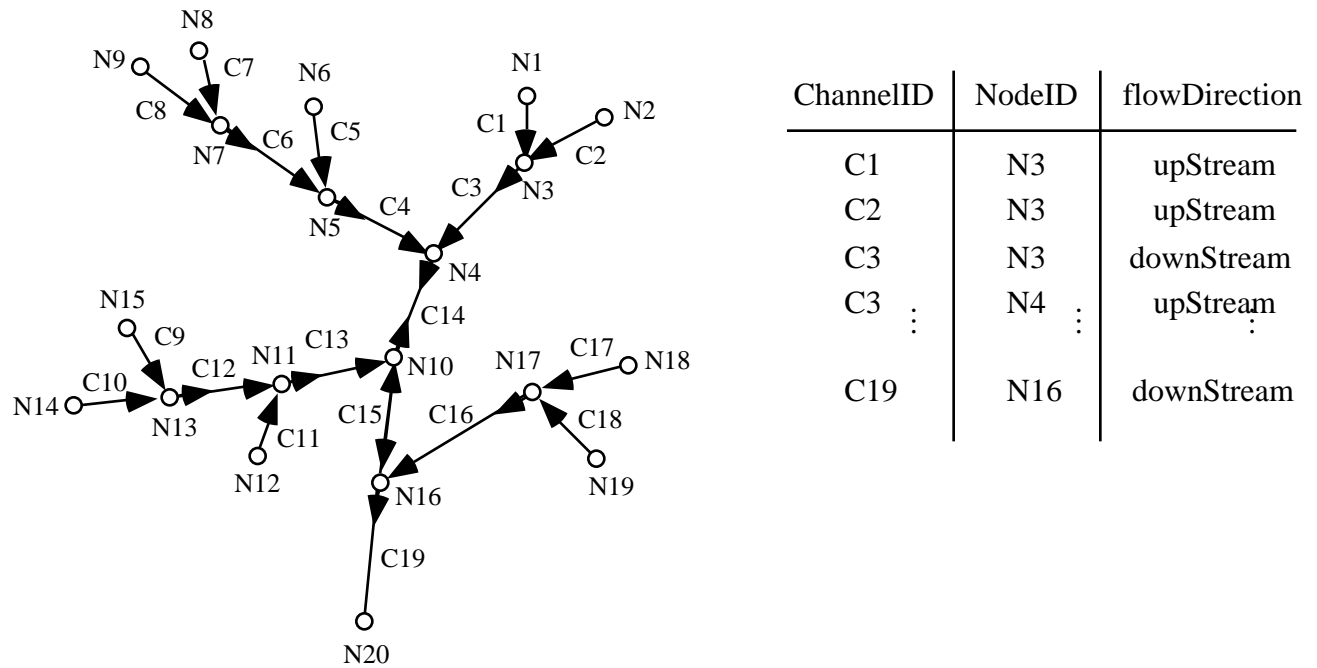


Figure 3: The network after applying the acute angle assumption at each junction.

3.2 Parent Channels

The parent channel at a junction is based on the clockwise and counter-clockwise angles from the downstream channel to the candidate channels (Figure 4). The following algorithm determines the parent channel P of a downstream channel D at a junction J .

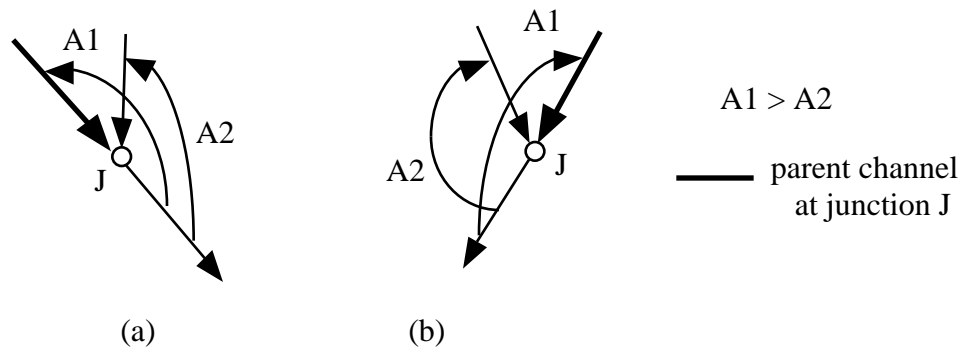


Figure 4: Parent channels at junction J : (a) counter-clockwise and (b) clockwise.

- All upstream channels U form the set of candidates c for J 's parent channel ($c \in \{U_{i=0..n}\}$).

- Calculate the angles $\theta_{D,c}$ from the downstream channel D to all candidate channels $C = c$, both clockwise $\theta_{D,c}^+$ and counter-clockwise $\theta_{D,c}^-$.
- Eliminate iteratively from the set of candidates the upstream channel with the smallest angle until only one candidate channel is left. This is the parent channel.

```

while cardinality( $c$ ) > 1 do
     $c = c \setminus (C \text{ where } \min(\theta_{D,c}))$ 
return  $P = \{C, c\}$ 
    
```

The result of this step is a set of channel pairs, describing the parent channel for each channel.

3.3 Identification of Branches

Each set of adjacent channels represents a branch. The goal is to identify first the possible start or end channels, and then the other adjacent channels that form a branch.

3.3.1 Possible Start or End Channels

The set of possible start and end channels of the branches in a network is the union of two types of channels:

- All external channels whose external nodes have in-degree 1 and out-degree 0 (e.g., channel C19 in Figure 3). These external nodes of such channels represent *potentialdestinations* (e.g., N20) and are used as start nodes of the respective branches.
- All internal channels that are the downstream channels at one junction—called *source-outlet junction*—and whose upstream channels have nodes of out-degree 1 and in-degree 0. Those nodes represent *potentialsources*. For example, channel C12 in Figure 3 is such an internal channel. Node N13 is a source-outlet junction, while nodes N14 and N15 are potential sources.

The result of this step is a set of channels that qualify as start or end channels of the branches of the network, and the potential destination nodes or source-outlet junctions are the start nodes of the branches.

3.3.2 Adjacent Branch Channels

The goal of this step is to identify for each possible start or end channel their adjacent channels. A branch B_a is formed for each start or end channel by recursively adding the corresponding adjacent channels. For each branch, the current channel C is initialized with the start or end channel. The current node N_1 is set to the last node of the branch, which is the node of C that is different from the potential destination node or source-outlet junction.

```

branches: SE { $B_a$ }
 $a$  0
for each  $C$  SE do:
   $a$   $a + 1$ ;  $B_a$   $C$ ;  $N$  lastN( $B_a$ ); adjacentBranch ( $B_a$ ,  $C$ ,  $N$ )
end branches

```

The operation *adjacentChannel* determines for the current channel C the next branch channel that shares the common node N , adds that channel to the branch B_a , and recursively applies *adjacentChannel* for the channel found. The end of the branch is reached if (1) the current node N is a potential destination or a source-outlet junction; (2) N is a junction in which channel C is upstream to N and a tributary to the downstream channel, and the parent channel of C is internal; or (3) N is a junction in which C is downstream of N and N represents a source-outlet junction. For a junction N and the current channel C being upstream of N , the next branch channel is the downstream channel D^N at N if C is the parent channel of D^N ; or if C is not the parent channel of D^N and the parent channel is an external channel. For a junction N and the current channel C being downstream of N , the next branch channel is the parent channel of C if all upstream channels at N are internal channels; or it is the tributary channel of C if the parent channel of C is an external channel.

The following operations and notations are used in the algorithm *adjacentChannel*: $\text{nodeType}(N)$ determines the type of node N (*potential_destination*, *source_outlet*, or *junction*), and $\text{channelType}(C)$ returns the type of channel C (*internal* or *external*). U^N and D^N stand for the upstream and downstream channels of node N , respectively. $P(C)$ and $T(C)$ find respectively the parent channel and tributaries of channel C .

```

adjacentChannel:  $B_a, C, N \rightarrow B_a$ 
if  $\text{nodeType}(N) = \text{potential\_destination}$  or  $\text{nodeType}(N) = \text{source\_outlet}$ 
  then return  $B_a$ 
if  $\text{nodeType}(N) = \text{junction}$  then
  if  $C \in U^N$  then
    if  $C \in T(D^N)$  and  $\text{channelType}(P(C)) = \text{internal}$ 
      then return  $B_a$ 
    if  $C = P(D^N)$ 
      then  $B_a \leftarrow B_a + D^N; C \leftarrow D^N; N \leftarrow \text{lastN}(B_a); \text{adjacentChannel}(B_a, C, N)$ 
    if  $C \in P(D^N)$  and  $\text{channelType}(P(C)) = \text{external}$ 
      then  $B_a \leftarrow B_a + D^N; C \leftarrow D^N; N \leftarrow \text{lastN}(B_a); \text{adjacentChannel}(B_a, C, N)$ 
  if  $C \in D^N$  then
    if for all  $u \in U^N : \text{channelType}(u) = \text{external}$  then return  $B_a$ 
    if  $u \in U^N : \text{channelType}(u) = \text{internal}$ 
      then  $B_a \leftarrow B_a + P(C); C \leftarrow P(C); N \leftarrow \text{lastN}(B_a); \text{adjacentChannel}(B_a, C, N)$ 
    if  $\text{channelType}(P(C)) = \text{external}$ 
      then  $B_a \leftarrow B_a + T(C); C \leftarrow T(C); N \leftarrow \text{lastN}(B_a); \text{adjacentChannel}(B_a, C, N)$ 
end adjacentChannel

```

The set of branches $\{B\}$ must be normalized to eliminate duplicate branches that were generated in opposite directions:

```

for each  $B_i \in \{B\}$  do:
   $\{B\} \leftarrow \{B\} \setminus B_i$ 
  where  $(\text{startN}(B_i) = \text{lastN}(B_j) \text{ and } \text{lastN}(B_i) = \text{startN}(B_j) \text{ and } i \neq j)$ 

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The result is the set of branches and their corresponding channels (Figure 5). All internal channels must be associated with one branch.

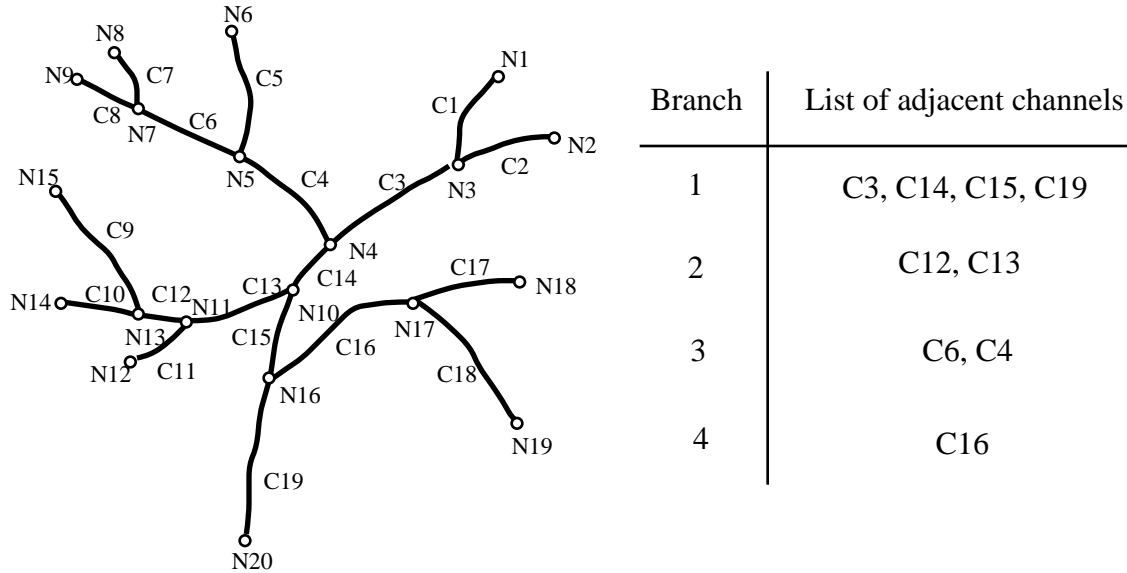


Figure 5: Network and its branches after applying the branch topology model.

3.4 Reasoning about Candidate Branches

Given the set of branches, we have to find from this set what branches are candidates for containing the outlet channel. Candidates are those branches that do not feed any other branch found, therefore, a candidate's external nodes (first or last node) must not be an internal node of another branch. Figure 5 shows branches B_2 , B_3 , and B_4 that feed branch B_1 ; therefore, neither B_2 nor B_3 nor B_4 are candidates for the branch with the outlet. On the other hand, channel B_1 does not feed any other channel and, therefore, is a candidate for the branch that contains the outlet.

With the candidate branches found, the last step is the identification of the external nodes of the branches in order to infer the probable outlet channel. Table 2 shows possible configurations of the external branch nodes and their probable outlet channel. If the candidate node is a potential destination then the outlet channel is the branch channel that contains this node. On the other hand, if the candidate node is a junction then the outlet channel is the parent channel of the branch channel that contains this node. When the first and last node of the candidate branch are of the same type (i.e., two destinations or two junctions), then the probable outlet is obtained by counting the number of channels with flow direction toward each node, and choosing as the outlet the node with the higher count. This decision is based on the assumption that the angle rules are efficient for more than 80% of the junctions.

branch nodes		probable outlet channel
first node	last node	
destination	junction	first branch channel
destination	destination	first or last branch channel
junction	junction	parent channel of first or last branch channel

Table 2: Configurations of external branch nodes and their probable outlet channels.

4 Implementation and Results

A program for automatic derivation of flow directions in river networks has been implemented in C++. The inference was fully automated and no user interaction was required. The program was applied to six dendritic drainage networks located in the Kentucky region, the same data sets that were used by de Serres and Roy [3] to verify the efficiency of the angle rules. Table 3 summarizes our empirical results. In all but one case the correct outlet was found. Despite a relatively high number of incorrectly classified junctions, the branch topology model found the correct candidate branch and candidate node for all networks. Only in one case (Monterey-Mo5) was an incorrect outlet channel selected from among the candidate nodes. In this case the candidate node represents a junction, and due to geologic control the 180° -assumption for the junction geometry was not satisfied, such that an incorrect parent channel was inferred; however, all of the upstream channels from this junction were associated with the right flow direction.

Kentucky Networks [3]	number of junctions	Angle junction rules			Branch topology model			
		correct junctions	incorrect junctions	% of correct junctions	% of correct junctions	candidate branch	candidate node	outlet channel
Inez-In1	65	52	13	80%	100/0	correct	correct	correct
Inez-In2	50	42	8	84%	100/0	correct	correct	correct
Monterey-Mo1	65	61	4	94%	100/0	correct	correct	correct
Monterey-Mo2	23	17	6	74%	100/0	correct	correct	correct
Monterey-Mo3	84	77	7	92%	100/0	correct	correct	correct
Monterey-Mo5	97	90	7	93%	99/1	correct	correct	wrong

Table 3: Results of the branch topology model on the Kentucky networks.

We also tested the algorithm for a river network in which the main channel is much wider than the tributaries and, therefore, represented as an areal feature. The left bank and the right bank together with their affluent channels were digitized as separate networks, and each sub-network was analyzed separately. The test area was Rio São Manuel, located in Alta Floresta north of Mato Grosso, Brazil, and was digitized from a 1:100,000 map of the Brazilian Institute of Geography and Statistic (Figure 6). For both networks, the outlet was derived correctly (Table 4).

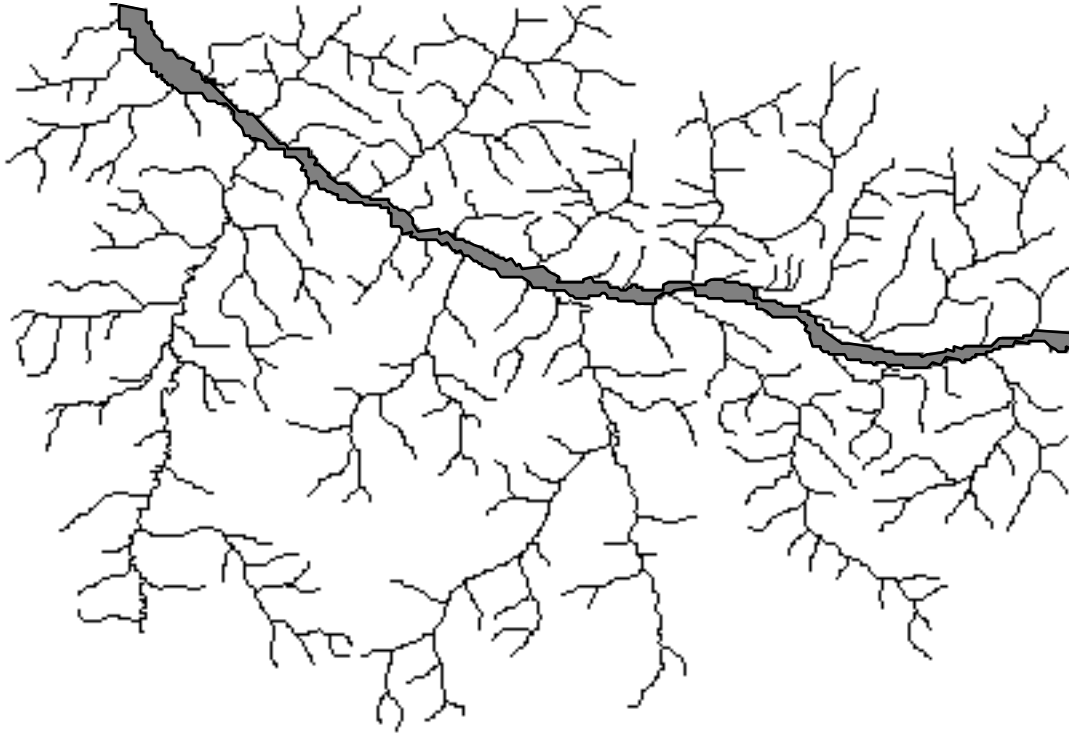


Figure 6: Rio São Manuel, digitized from 1:100,000 map of the Brazilian Institute of Geography and Statistic, reference number SC.21-X-C-V MI-1559. The outlet is in the upper left corner.

Rio São Manuel	number of junctions	Angle junction rules			Branch topology model			
		correct junctions	incorrect junctions	% of correct junctions	% of correct junctions	candidate branch	candidate node	outlet channel
right bank	93	93	0	100%	100/0	correct	correct	correct
left bank	144	135	9	94%	100/0	correct	correct	correct

Table 4: Results of the branch topology model for Rio São Manuel (Figure 6).

5 Conclusions

The inference of the flow direction of river networks is an important step in the development of formal methods for reasoning about geographic space [4]. This paper described an algorithm that uses heuristics about the angles at the junctions of river channels, rather than elevation data to reason about the channel's flow directions. The algorithm is based on the identification of branches of channels and their connectivity. This branch topology model is an appropriate inference method for networks in flat terrain. In these cases, without information from a digital elevation model, the flow directions can be determined using just the planimetric representation with its topological and metrical properties about connectivity and junction angles. Results obtained for dendritic drainage networks from the Kentucky region have shown the robustness of the algorithm. The tests showed that the model finds the correct outlet channel, even with inconsistency of flows after the application of angle rules on each network junction.

The branch topology model has a very practical application in the realm of building geographic information systems. The Amazon region, located in northern Brazil, extends over 5,000,000 km² with approximately 100 m elevation difference along large parts of the Amazon and Solimoes rivers. The small elevation differences cannot be surveyed from the air and obstacles impede access to the surface. Current efforts in building a geographic information system of this area to monitor deforestation [1] face the difficulties of covering a very large area with no existing maps and many temporal changes caused by high-water and erosion [12].

6 Acknowledgments

We are grateful to André Roy who provided his test data sets from his previous experiments. Thanks also to Andrew Frank, Mike Goodchild, David Mark, and Renato Barrera for their comments and suggestions.

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