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Calibrating the Meanings of Spatial Predicates from Natural Language: Line-Region Relations[*]

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Abstract

Results from human subjects testing are used to calibrate the meaning of “the road crosses the park” and three other similar sentences in English. Sixty stimulus maps represent two or more examples of each of the 19 line-region spatial relationships defined by the 9-intersection model. The subjects’ mean agreement that the sentence applies to each map is expressed as a weighted sum of binary variables that define which of the 9-intersection categories the relationship between road and park on the map belongs to. Statistically, regression equations based on 9-intersection topology alone account for between 60 percent and 90 percent in the variation in mean subjects’ responses. “Crosses” and “goes across” appear to be more sensitive to variations in metric properties that are “goes through” and “enters.” Results confirm that the method used has potential for defining cognitively meaningful spatial predicates and for comparing the meanings of similar terms in different natural languages.

1. Introduction

Many spatial queries and spatial tasks include constraints in the form of spatial relations that must be satisfied. In problem solving and spatial reasoning, such constraints would commonly be stated in natural language—some examples in the English language are: hazardous waste sites *on* flood plains; schools *near* railway tracks; pipelines that *cross* rivers; and industrial land *next to* railways. When predicates representing spatial relations are included in a spatial query language of spatial modeling system, they are commonly given names. They also need formal definitions to be processed in a spatial database,

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because such spatial relations are usually derived from a geometric or topological data model, rather than being recorded explicitly. It seems obvious, yet it is seldom stated, that use errors will be reduced if the names chosen for such spatial predicates match the common-sense meanings of those terms. And it also seems obvious that the set of spatial predicates presented to the user should include the concepts that users include in their queries or models. Whereas it is possible to model spatial relations formally first, and then name them later, an alternative approach seems more likely to satisfy the principles noted above. This approach is to examine the meanings of spatial (locative) terms in natural language, and then to formalize these meanings to produce a suite of predicates for use in queries and models.

The 9-intersection model of spatial relations (Egenhofer and Herring 1991)—and its predecessor the 4-intersection (Egenhofer and Franzosa 1991; Egenhofer and Herring 1991)—have been used extensively in the GIS and spatial database community (Pigot 1991; Clementini *et al.* 1993; Papadias and Sellis 1993; Wazinski 1993), and implementations in research prototypes (de Hoop and van Oosterom 1992), in macro languages (Hadzilacos and Tryfona 1992; Mark and Xia 1994), and in commercial systems (Herring 1991) already exist. While most of these studies have focused on the formalists' perspective on spatial relations (how to implement, extend, or formally simplify the set of relations), there have been only a few attempts to examine the links between such formal definitions and the use of spatial predicates in natural language (Mark and Egenhofer 1992).

This paper presents new results from such experimental work, which demonstrate that the *interplay* between formal mathematics and human subjects testing is of considerable value in the search for fundamental theories of spatial relations. These investigations employ human-subject testing, a research methodology that is complementary to the development of formalizations with mathematical tools. A basic assertion in this paper is that such human-subjects experiments can guide mathematicians and software engineers as to which distinctions among topological and metric details are worth making when defining the semantics of terms for spatial relations to be used in GIS query languages, and which are not.

One major goal of this work is to provide an answer about the granularity of natural-language spatial relations. Specific research questions include:

- Are the fundamental aspects of the 9-intersection—empty and non-empty intersections of interiors, boundaries, and exteriors—the critical parts for the definition of natural-language spatial predicates?
- Do humans in natural (English) language make use of all the details distinguished by the 9-intersection?

2. The 9-Intersection

The relations of concern in this paper are those between objects whose geometries are represented by a simple line and a region, respectively. A simple line has exactly one start- and one end-node; its interior does not self-intersect nor do the start- or end-nodes coincide with the line's interior or with each other. A region is a homogeneously 2-dimensional object whose boundary forms a Jordan Curve. Such lines and regions are assumed to be embedded in the Cartesian plane $R^{2.}$

The 9-intersection defines binary topological relations in terms of the intersections between the interiors, boundaries, and exteriors of the two objects of concern (Herring 1991; Egenhofer *et al.* 1993). Each of these intersections is classified as being empty or non-empty, giving rise to 512 different relations that can be distinguished by this model. The 9-intersection model is generally applicable to the most common representations used for geographic data such as points, lines, and regions. The empty/non-empty intersections that can be realized for a particular combination of points, lines, and regions depend on topological properties of such objects, e.g., whether their boundaries are Jordan Curves or not; whether the interior is connected or disconnected; or whether the exterior is a separation or not. The 9-intersection distinguishes 19 different topological relations between a line and a region.

3. The Experiment

The stimuli for the experiment were 60 identical drawings of a region representing a state park, and for each of these drawings, a road with a different position relative to the park and sometimes also with a different shape (see examples in Figure 1). The first 38 stimuli were two examples for each of 19 topologically-distinct cases distinguished by the 9-intersection model. An additional 22 stimuli were designed to explore probable influences of geometric configuration (shape, position) of the region and the line on specific spatial relations. In addition to the road and park, these maps also had common "base map" information such as towns, rivers, lakes, and other roads, and a thin road connected the test road to the base-map roads in each case. These stimuli were printed in black-and-white.

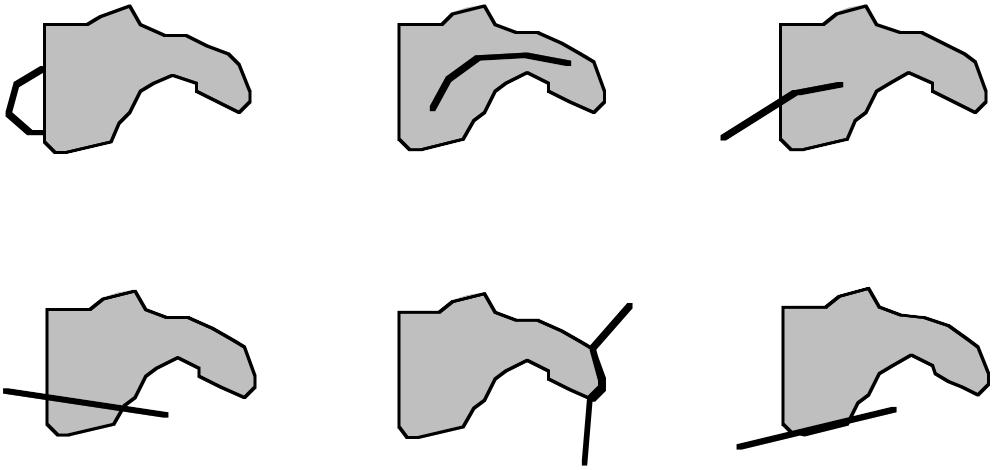

Figure 1: Some examples of the roads and parks from the stimuli used in this research. The right and middle examples in the lower row are topologically identical but geometrically distinct. The actual stimuli added identical base map information to each diagram.

Each subject was presented with a sentence in English, describing the spatial relation between a “road” and a “park.” Subjects were asked to compare their sentence to each of the 60 diagrams, and to evaluate on a scale of (a) to (e) the strength with which they disagree (a) or agree (e) that the sentence describes the situation portrayed in that diagram. Test instruments were distributed during a class period, and the students were told that their participation was entirely voluntary. Data on “the road crosses the park,” “the road goes across the park,” and “the road goes through the park” were collected from students in an introductory university-level course on world regional geography. From this group, we obtained 31 responses to “crosses,” 28 to “goes across,” and 28 to “goes through.” Data also were collected on the sentence “the road enters the park” from students in Geography 120 ("Maps and Mapping”), with 33 responses being obtained. In analyzing the data, it became apparent that almost certainly, some subjects had reversed the agree-disagree scales; and objective procedure was developed to detect these “reversers.” One subject was eliminated for “across,” two for “cross,” two for “through,” and two for “enters.”

After following the above procedures, data for the remaining subjects were converted to numerical scales, averaged across subjects for each stimulus, and then rescaled so that unanimous strong disagreement would yield a value of 0.0, and unanimous strong agreement would be 1.0. These mean agreement indices are similar to membership functions for fuzzy sets, and future work may apply fuzzy set theory to these and similar data; for now, however, these numbers are just considered to be measures of strength of agreement, and will be called “membership values.”

4. Comparison of Different Predicates

The results for the four sentences tested are very encouraging regarding the method. For "the road crosses the park," mean membership values ranged from 0.000 to 0.983. For "across," the values ranged from 0.009 to 0.894, for "goes through" they ranged from 0.038 to 0.885, and for "the road enters the park" from 0.008 to 0.855. Thus the subjects' responses to the stimuli come close to spanning the entire range of possible responses, namely from 0 to 1. Also, confirming common sense intuitions, all the minimum memberships for these predicates were for cases in which the road was completely disjoint from the park.

To examine the meanings of these sentences further, we first plotted membership values for the predicates against each other. The sentence "the road crosses the park" was selected as a basis for the comparative analyses, in part because is intuitively more basic that "goes across" or "goes through," and in part because it had the greatest range of mean responses. Thus, we plotted results for each of the other three sentences against results for "crosses."

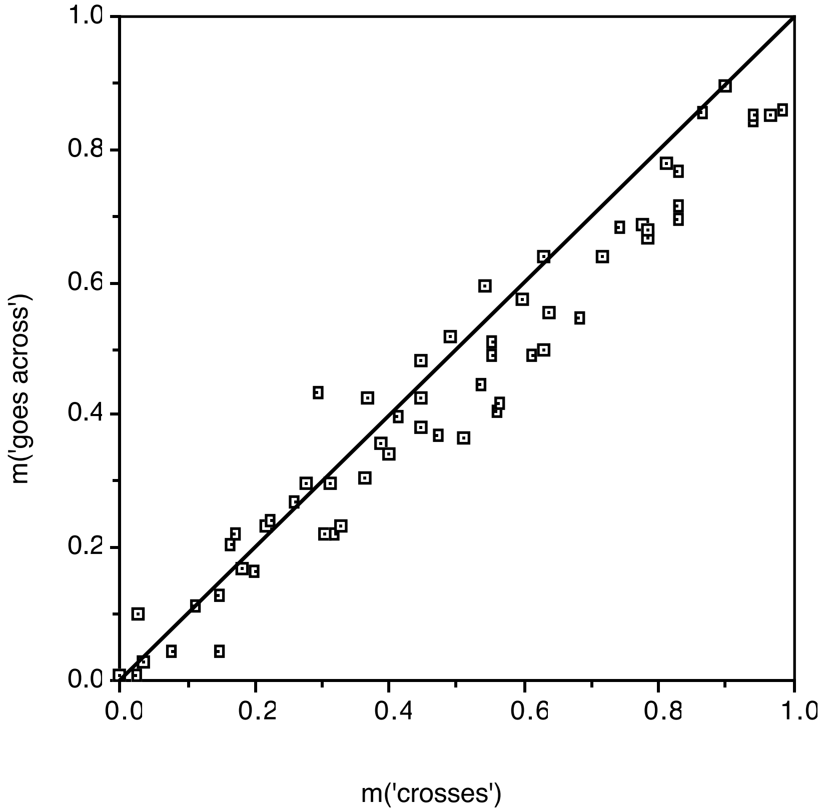

Figure 2: Plot of m("goes across") against m("crosses").

Figure 2 shows that the meanings of “the road crosses the park” and “the road goes across the park” are about the same. This was expected, as these sentences were selected to test what the method would reveal for “synonymous” sentences. However, although there is no systematic departure from linearity, it is interesting and somewhat surprising to see that memberships in “goes across” are lower than they are for “crosses” in 46 of 60 cases. Could the somewhat lower values be due to the slight difficulty of judging a somewhat dynamic sentence (“goes across”) from static diagrams? Larger sample sizes will be needed to confirm whether there really is a difference between the meanings of “crosses” and “goes across” for roads and parks.

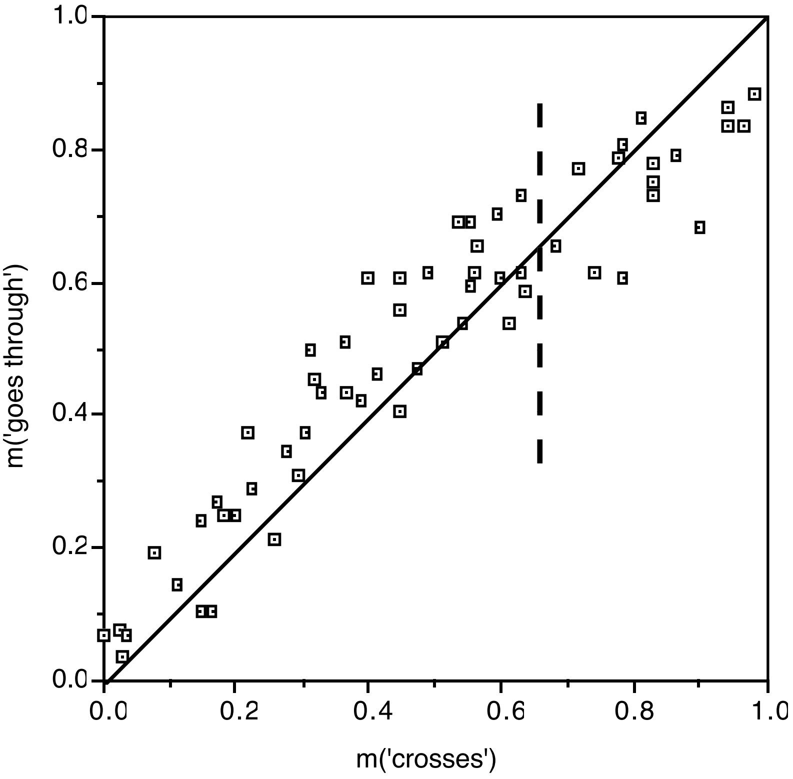

Figure 3: Plot of m(“goes through”) vs. m(“crosses”)

The relationship between mean memberships in “goes through” and “crosses” is more complicated (Figure 3). As expected, the over-all pattern is one of similarity. However, the residuals are much more systematic than in the previous example. Membership in “crosses” exceeds that for “goes through” in 12 of the 16 cases for m(“crosses”) > 0.66 (to the right of the dashed vertical line in Figure 3); on the other hand, membership is higher for “goes through” in 35 of the 44 cases with relatively lower memberships in “crosses.” To put this another way, in cases where the road strongly crosses the park, then it goes through to a similar, yet weaker extent; whereas if membership in both predicates is lower, “goes through” tends to be stronger, perhaps because “goes through” is not as dependent upon geometry as is “crosses.” The effects of geometry on these sentences can perhaps best be illustrated by plotting memberships in “goes through” against "crosses" for six cases with the same topology (Figure 4). Due to variations in geometry, as superimposed on the scattergram, values for "cross" range from 0.982 to 0.319, whereas values for "goes through" had a smaller range, from 0.885 to 0.452. The two spatial relations may have the same prototype, but geometric departures from that prototype (stimulus #58 in the diagram) reduce membership in "crosses" more than in "goes through."

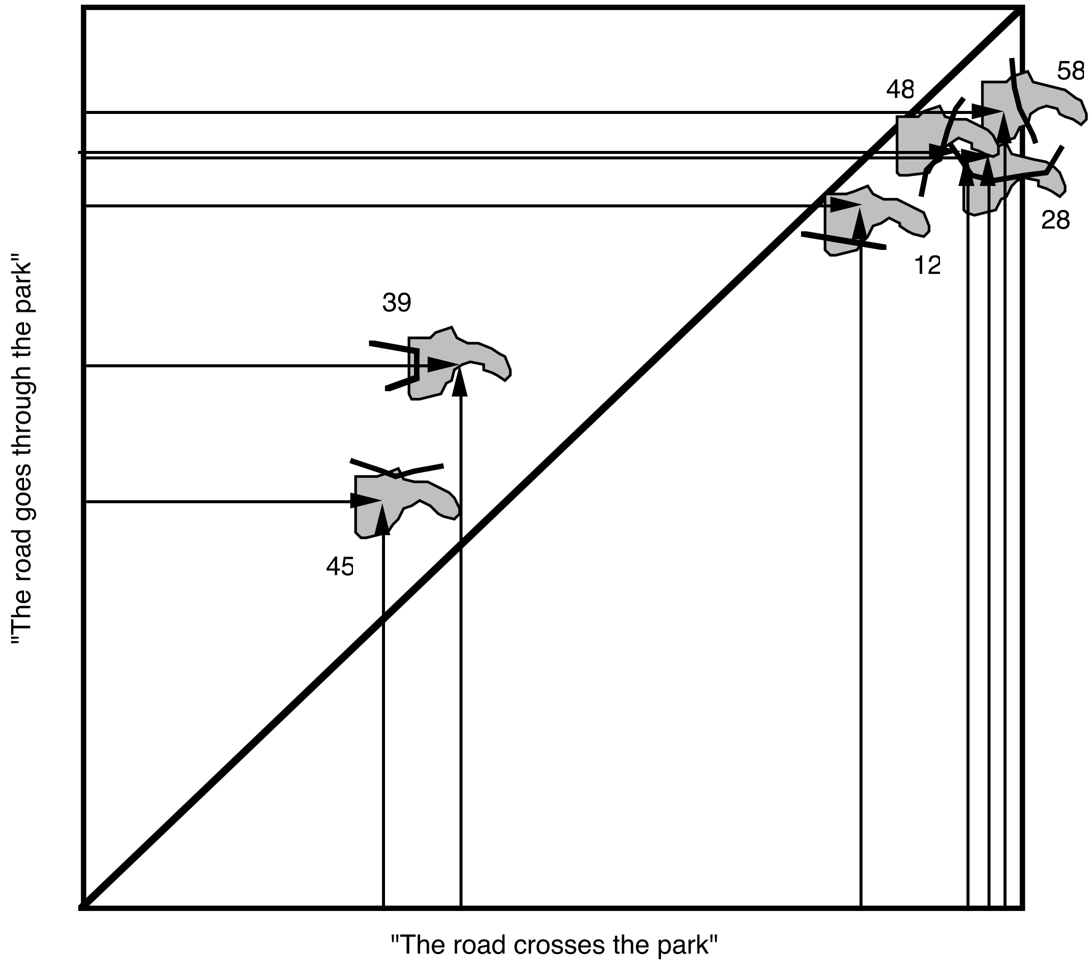

Figure 4: Membership in "the road goes through the park," plotted against "the road crosses the park." In particular, note that stimulus #39 has a stronger rating for "goes through" than for "goes across."

"Enters" has quite a different meaning than does "crosses," although there is some similarity (Figure 5). The majority of the stimuli had similar, and relatively high, memberships in "enters." The cases for which "enters" values were low also had low values for "crosses," because both seem to require an interior-interior intersection. In fact, there is a perfect dichotomy of values of "enters" in relation to this topological determinate: stimuli with no interior-interior intersection had "enters" values between

0.008 and 0.427, whereas those for which the interior-interior intersection was not empty ranged from 0.508 to 0.855

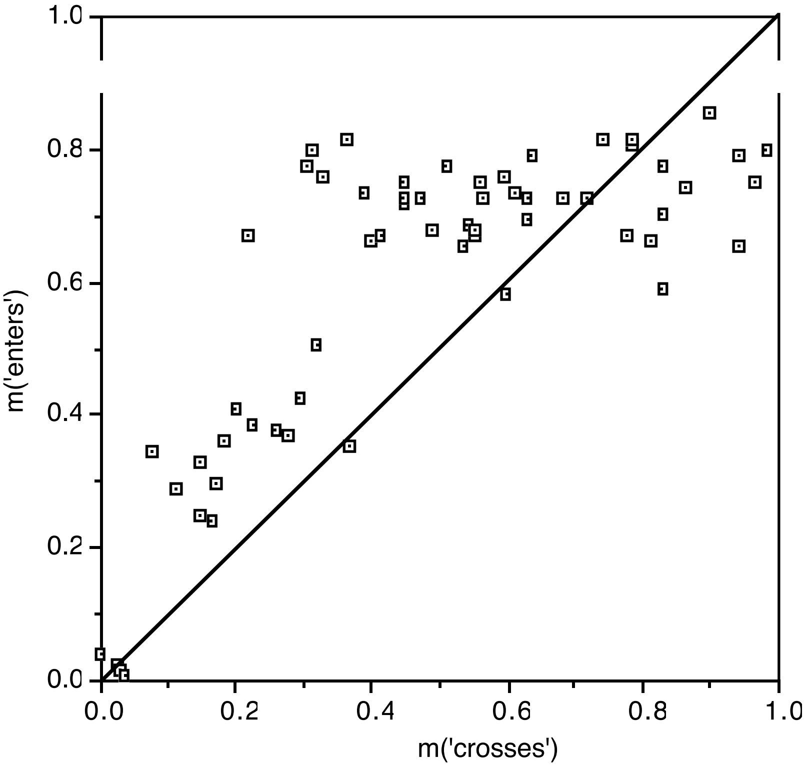

Figure 5: Plot of m("enters") vs. m("crosses").

5. Regression Analysis

To explore further the relationships between the subjects' responses, and the 9-intersection model, we conducted a multiple regression analysis using MINITAB. For each of the four sentences, we regressed the mean agreement rating on six binary variables, representing the existence or non-existence of intersections between the ends and body of the line with each of the three parts of the region. Since these six "bits" completely characterize the 9-intersection categories, the variance explained by the model is a measure of how well the mean response of subjects can be predicted from topology alone, as characterized by this model. Any unexplained variance can be attributed to geometric variations, effects of the graphic designs or the test sequence, or to sources of "random" variation. As noted above, the first 38 stimuli represented 2 examples of each of the 19 spatial relations—they were designed arbitrarily, but with the intention that they be "typical" cases. Many of the last 22 stimuli, however, were added to explore subjects' responses to specific kinds of variation in geometry, especially in relation to the "cross" relation. Therefore it seemed appropriate to compute regression equations separately for just the first 38 stimuli, as well as for all 60 stimuli.

Table 1 shows the regression coefficients for four sentences tested. (The goodness-of-fit measures for these equations are given in Table 2 and discussed below.) In the equations, the variables have 2-letter names, in which the first letter denotes a part of the line (I = Interior, the "body" of the linear feature; B = Boundary, the end nodes of the line), and the second letter denotes a part of the region (I = Interior; B = Boundary; E = Exterior). The variables were coded as "1" if the particular intersection existed (i.e., was non-empty), and "0" if it was empty. For the 9-intersection between lines and regions in 2-D space, the line's exterior must intersect with all parts of any region, and hence the EI, EB, and EE variables are omitted from this analysis.

1. Regressions for First 38 Stimuli

across = 0.390 + **0.487 I I** + 0.041 IB - **0.133 I E** - **0.327 B I** - 0.080 BB - 0.128 BE

cross = 0.369 + **0.576 I I** + 0.011 IB - **0.158 I E** - **0.352 B I** - 0.060 BB - 0.056 BE

through = 0.335 + **0.514 I I** + 0.042 IB - **0.165 I E** - **0.210 B I** - 0.020 BB + 0.001 BE

enters = 0.177 + **0.363 I I** + **0.117 I B** + 0.006 IE + 0.033 BI + **0.063 B B** + 0.015 BE

2. Regressions for All 60 Stimuli

across = 0.340 + **0.419 I I** + 0.066 IB - **0.169 I E** - **0.192 B I** - 0.075 BB - 0.018 BE

cross = 0.278 + **0.533 I I** + 0.046 IB - **0.161 I E** - **0.213 B I** - 0.036 BB + 0.032 BE

through = 0.320 + **0.484 I I** + 0.065 IB - **0.187 I E** - **0.179 B I** - 0.017 BB + 0.032 BE

enters = 0.148 + **0.365 I I** + **0.140 I B** + 0.006 IE + **0.052 BI** + **0.061 B B** + 0.022 BE

Table 1: Coefficients of regression equations for agreement ratings against 6 binary variables that represent the topology according to the 9-intersection. Coefficients that are statistically-significant (0.05 level) are presented underlined and in bold type; significance test did not take into account the numbers of subjects in the degrees of freedom.

Since the pattern of coefficients is very similar for the N=38 and N=60 regressions, attention here will be focused on the data for the first 38 stimuli, since as just noted, many of the last 22 stimuli were geometric special cases of various kinds. One immediate result is that the patterns of significant coefficients are identical for "across," "cross," and "through." Basically, for roads and parks at least, these three spatial relations mean about the same, confirming our intuitions. Further interpretation of the regression equations thus will emphasize the more basic term of this set, "cross." The largest coefficient is for II. That means that, other things being equal, a case in which the road is at least partially within the park will have a higher membership in "the road crosses the park" than would a case in which the road did not go into the park at all. This simply confirms the obvious. The coefficient with the second highest significance level is BI, and its value is -0.352. This means that if one or both ends of the road are inside the park, the expected membership in "cross" drops sharply. This confirms quantitatively and with a larger sample size Mark and Egenhofer's (1993) conversion of Talmy's (1983) interpretation of the meaning of "cross" into the 9-intersection model. The third coefficient that is significant is in all three cases IE. This means that, other things being equal, if the road goes outside the park at all, subjects would give it a somewhat lower rating of agreement that it "crosses" the park. The interpretation of this result is unclear. Figure 6 shows observed and expected values from the regression over the first 38 stimuli, with the additional 22 stimuli plotted for reference.

Also confirming intuitions, "enters" has a different topological meaning. As for the "cross" group, "enters" is also most strongly related to whether the road is at least partially inside the region (II). However, the additional two significant coefficients are IB and BB, different from IE and BI for "cross," and furthermore, these coefficients are positive, not negative. IB means that the road's body either intersects or is co-linear with the park boundary. Intuitively, this is very close to the essence of what "the road enters the park" means. If both IB and BB are true, the predicted membership in "the road enters the park" would be 0.657 or higher. Lastly, there is a slight but significant increase in the expected membership in "enters" if an end of the road lies on the boundary (BB); as with the third-ranked significant coefficient for "crosses," the meaning of this is not evident.

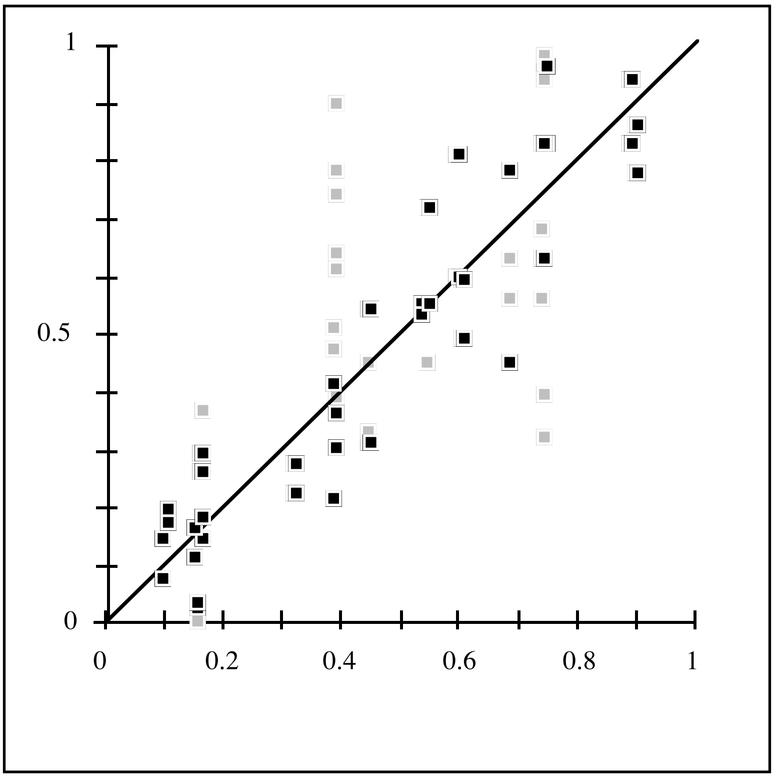

Figure 6: Observed (Y) vs. expected (X) agreement with “the road crosses the park,” where the expected values are based on a regression of observed values on six binary variables representing the 9-intersection topology. The 38 stimuli used to calibrate the regression equation are showed in black, and the additional 22 stimuli are shown in gray.

The over-all proportion of variance explained by topology alone is perhaps as interesting as the equations and coefficients. As summarized in Table 2, the results confirm that topology alone can account for the majority of the variation of mean subject responses to all four sentences tested. For the regressions over just the first 38 stimuli, between 84 and 91 percent of the means[1] of subjects’ responses is accounted for by the 9-intersection topology.

	N=38	N=60	difference
goes across	84.8	60.6	24.2
cross	86.3	68.8	17.5
goes through	90.7	84.2	6.5
enters	89.3	89.5	-0.2

Table 2: Proportion of variance in mean subject responses that is “explained” by the binary topological variables of the 9-Intersection model.

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[1] It is important to note that this is not 84-91 % of the total variation; most individual variation has been removed through the process of averaging for each stimulus the responses of about 30 subjects.

Table 2 also shows the proportion of variance explained for the regressions over all 60 stimuli, and the difference between these N=38 and N=60 regressions. Except for "enters," where the results are about the same for both sets of stimuli, the regression fit is stronger over the first 38 stimuli (2 for each of the 19 spatial relations) than over the entire 60. This was expected, because many of the extra 22 stimuli were added to represent geometric special cases, especially with regard to how geometry might influence the meaning of "the road *crosses* the park." Not surprisingly, the drop is greatest for "goes across" and "crosses," with "goes through" being quite a bit more strongly explained by topology alone when computed over the 60 stimuli. Evidently, geometric variations introduced to change responses regarding "crosses" did exactly that, but did not substantially alter the strongly topological nature of responses to "goes through" and "enters." This appears to confirm the intuition that "cross" has a more geometric component to its meaning than does "goes through." This was pointed out by Talmy (1983), who noted that "cross" involves a path from one side of the region to the other—and "side" implies a geometric property of the region. On the other hand, "goes through" seems to just imply penetration in and out again, a more purely topological relation. This can be illustrated more clearly with other figures and grounds, as the following introspections suggest.

Linguists often study language through introspective examination of their native languages. This method can be applied here to further examine the meanings of "through" and "across." As noted above, and as supported by empirical evidence, the following pair of sentences would seem to mean about the same:

- The road goes through the park.
- The road goes across the park.

The near-identity of "through" and "across" also seems to hold if we are talking about a man walking though or across a field. There may be a slight shade of difference in the meanings of the next pair of sentences, but again, the meanings seem very similar:

- The highway goes through Boston.
- The highway goes across Boston.

But now consider this pair of sentences:

- When we went to Seattle, we drove through Canada.
- When we went to Seattle, we drove across Canada.

The first sentence would be true if any part of the route is in Canada. It would, for example, be reasonable to say that if we had driven from Boston to Buffalo, then cut through southern Ontario, and then driven on U.S. roads from Detroit to Seattle. However, the second sentence implies that most of the route was in Canada, that is, from one "side" of Canada to the other.

It seems that "through" implies a three-dimensional Ground object, or perhaps a medium, whereas "across" implies that the Ground is a surface (two-dimensional object). For some features at a geographical scale, either a 2-D or 3-D schema may be applied to the same entity in the world. A park of a field can be conceptualized as either a 2-D surface or a medium, and for a road crossing a park or a man crossing a field, the change of image-schema (Johnson 1987; Mark 1989) required as one switches between "cross" and "through" seems to produce little if any shift in meaning. However, for a country, going "through" it seems to imply any path that enters and then leaves (pure topology), whereas going "across" it implies a side-to-side (east-west) path (topology plus geometry). The experimental results seem to confirm that this effect is present to some extent even for the road and the park (Figure 4).

From examples like these, it seems clear that topology and geometry alone will not be sufficient to connect words and meanings for spatial relations in all cases. There will be at least some effect of the semantics, of the nature of the FIGURE and GROUND objects. For this reason, we decided to restrict our early experiments to cases where the FIGURE is a road and the GROUND is a state park. We recognize that by avoiding the added complications of the effects of different kinds of FIGURES and GROUNDS, we are at the same time producing results that apply only to those and similar phenomena, and that cannot be generalized automatically to all cases of FIGURES that are lines and GROUNDS that are regions. Obviously, there is much room for further work on this topic. And the idea that the meaning of a spatial predicate might depend on the attributes of objects as well as their topological and metric relationship presents a potential challenge to designers of spatial query languages.

6. Summary

In the introduction to this paper, we posed two specific research questions about the relationship between the 9-intersection model and humans' use of natural language spatial predicates. The answer to the second question, "Do humans in natural (English) language make use of all the details distinguished by the 9-intersection?" seems clearly to be "Yes." Although we have reported results of tests of just four sentences, all in the English language, the regression analyses of meaning membership on the six binary variables showed statistically significant regression coefficients for five of the six variables, three for the "cross/across/through" group, and two additional variables for "enters." Only the binary variable representing existence of an end of the road outside the park was never significant in these regressions, and it would almost undoubtedly be significant in the case of sentences like: "the road ends outside the park." This confirms with human subjects' data the formal analytical results presented by Egenhofer *et al.* (1993) showing that the added complexity of the 9-intersection model over the 4-intersection is compensated for by increased explanatory power.

The answer to the first question we posed, "Are the fundamental aspects of the 9-intersection the critical parts for the definition of natural-language spatial predicates," would clearly be "Yes" without the word "the" just before "critical parts." The evidence from the data presented here, and our other experiments with the 9-intersection, clearly indicate that it provides *a* critical part of computational models of spatial predicates in language, and perhaps the most important part. But the difference between the regressions for “cross” over the 38 and 60 stimuli clearly indicate that topology alone is not always the entire picture. Metric properties of a spatial relation have some effect on the meaning of “the road crosses the park,” and will no doubt dominate the meaning of other sentence, for example “Orono is near Bangor.” But the 9-intersection is clearly an excellent basis for defining the meanings of many natural language spatial predicates involving lines and regions, and we expect that the model will also be useful for relations between spatial features of other dimensions.

7. Acknowledgments

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