

The Arrow-Semantics Interpreter

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People often sketch diagrams to facilitate their communication. If computers understood such diagrams, people could interact with information systems more intuitively using similar multi-modal dialogs that include sketched diagrams. It remains a challenging problem, however, to make computers interpret arrow symbols, which are a frequent and versatile ingredient of such diagrams. This paper develops an algorithm for deducing possible semantic roles of arrow symbols. The *Arrow-Semantics Interpreter (ASI)* analyzes the elements that surround an arrow symbol and their spatial arrangements. It distinguishes four different semantic roles of arrow symbols—*orientation*, *behavioral description*, *annotation*, and *association*—each with distinctive requirements on the arrangement of the surrounding elements. The combination of these properties, together with rules for optional elements, yields the potential semantic roles of each arrow symbol from the pattern of its surrounding elements. The assessment shows the ASI derives for each arrow symbol only 1.31 potential semantic roles, which include for 79% of the analyzed diagrams the correct answer. This result indicates that the patterns of elements around arrow symbols are highly useful for deriving the interpretation of arrow symbols.

Keywords: arrows, arrow symbols, diagram interpretation, sketching.

1 Introduction

People often sketch diagrams to increase the understanding of their verbal expressions, because such diagrams allow them to directly capture the spatial and structural characteristics of information. If computers understood such diagrams, people could interact with information systems more intuitively, for instance, by sketching diagrams with a pen device. A number of pioneering sketch-based computer systems have been developed (Oviatt, 1996; Egenhofer, 1997; Landay and Myers, 2001; Davis, 2002; Ferguson and Forbus, 2002), which have demonstrated that computational diagram understanding is a highly promising technology that will enrich human-computer interactions.

Arrow symbols are a frequent ingredient of such diagrams. One reason for the arrow symbols' popularity is their versatility—even though their shapes are extremely simple, they capture a large variety of concepts, such as direction, movement, order, transition, and association. Arrow symbols themselves, however, do not have any specific meaning apart from the context. They always provide the information about the other elements, which may be illustrated explicitly around the arrow symbol or inferred from the context. This function of arrow symbols has been called their *semantic role* (Kurata and Egenhofer, 2005b). Arrow symbols may have a variety of semantic roles, such as *labeling*, *specifying a direction*, and *illustrating a movement*. In order to correctly understand a diagram that contains arrow symbols, its readers have to deduce the semantic role of each arrow symbol. Such interpretations are usually easy for human adults, who have sufficient experience in diagrammatic communication. For sketch-based systems, however, the interpretation of arrow symbols remains a challenging problem, as highlighted by current sketch-based systems that impose some restrictions on the use of arrow symbols and make sketch-based interactions less natural. A key impediment is the polysemy of arrow symbols, which requires disambiguation in order to make sensible interpretations, for instance through carefully crafted context (Tversky *et al.*, 2007).

The semantic role of an arrow symbol obviously depends on to what elements the arrow symbol refers and how the arrow symbol refers to those elements (e.g., linking two elements with an arrow symbol or attaching it to one element). The combination of arrow symbols and the elements to which the arrow symbols refer is considered a syntactic unit, called an *arrow diagram*. The elements to which the arrow symbols refer are then called the *components* of the arrow diagram, which may be represented by icons, text labels, or specific positions in a background drawing, such as a map. This paper builds on the observation that possible semantic roles of arrow symbols in simple diagrams can be derived from the pattern of components' arrangement (Kurata and Egenhofer, 2005a).

In addition to the components and their arrangements, the visual appearance of arrow symbols, and context may also influence the semantic roles of arrow symbols (Tversky *et al.*, 2007). To focus the discussions, however, this paper

leaves out these visual factors, considering that they contribute to the judgment of the validity of interpretations, but not directly to the derivation of possible interpretations.

Preliminary work on the interpretation of arrow diagrams included an analysis of arrow symbols that yielded four major semantic roles (Kurata and Egenhofer, 2005a; 2005b) and a formalization of binary relations between arrow-like lines (Kurata and Egenhofer, 2006b; 2006a). While the relations contribute primarily to the analysis of complex, possibly nested diagrams, the semantic roles and their definitions play a much more dominant role in the semantic interpretation of diagrams with arrows. Our earlier work had a limited scope, however, as it considered only simple arrow diagrams with at most one component and concentrated the classification of the semantic roles of arrow symbols, while this paper exploits them as a guideline to derive correct interpretations. To achieve this goal, this paper develops the *Arrow-Semantics Interpreter* (ASI), an algorithm for deducing the semantic roles of arrow symbols from the arrangement patterns of the arrow's components and their categories. With the algorithm's assessment, this paper offers further evidence that the revised categorization of the semantic roles captures the anticipated intuition.

The remainder of this paper is structured as follows: Sections 2 and 3 respectively review studies on arrow symbols and sketch-based systems, highlighting how they deal with arrow symbols. Section 4 classifies the semantic roles of arrow symbols. Section 5 introduces the *individual structure*, which models the spatial arrangement of components around an arrow symbol. Section 6 identifies the requirements on the individual structures when arrow symbols are used for each class of semantic roles. Based on these requirements and rules for optional components (Section 7), we can determine potential semantic roles of arrow symbols from the pattern of the components' arrangements. Sections 8 and 9, respectively, introduce and evaluate the ASI. Finally, Section 10 concludes with a discussion of future work.

2 Studies on Arrow Symbols

Tversky (2001) defined an arrow symbol as *a special kind of line, with one end marked, inducing an asymmetry*. This definition highlights two essential features of arrow symbols: linearity and asymmetry. They contribute to the affordance (Gibson, 1979) of arrow symbols, prompting the diagram readers to move their attention from the tail side to the head side of the arrow symbol. Accordingly, if the arrow symbol connects two elements, these elements are naturally ordered. Also, if the diagram is mapped onto a physical or conceptual space, people naturally imagine a movement in this space. Thanks to these characteristics, arrow symbols have a large variety of semantic roles.

Semantic diversity of arrow symbols has attracted researchers' interest. Van der Waarde and Westendorp (2000) identified seven usages of arrow symbols in

pictorial user instructions: moving direction, physical change or transformation, indication of a dimension, labeling, focusing the attention, indication of a sequence, and a part of designed symbols. Similarly, Blaser (2000) identified four usages of arrow symbols in sketch maps: pointing north, indicating a path direction, indicating the direction of a view, and describing where a road leads to. Horn (1998) collected various usages of arrow symbols and schematized them in a tree graph. Westendorp (2006) speculated how the usages of arrow symbols have diversified, pointing out that the widespread use of pictorial instructions contributes to the recent diversification.

Another important characteristic of arrow symbols, which also contributes to their popularity, is that arrow symbols enable the illustration of dynamic information in a static diagram. Bertin (1983) claimed that arrow symbols are the most efficient and often the only formula for illustrating a complex movement. Monmonier (1990) highlighted the usefulness of arrow symbols for illustrating spatial diffusion of ideas, migrations of tribes and refugees, advances of armies, and so forth. Tversky *et al.* (2000) demonstrated that the presence of arrow symbols encourages people to read causal and functional information from diagrams. Reversely, Heiser and Tversky (2006) demonstrated that people spontaneously use arrow symbols to indicate functional processes in diagrams when they are given functional descriptions.

3 Arrow Symbols in Current Sketch-Based Systems

Over the last ten years, a variety of sketch-based systems have been developed, aiming at more natural and effective human-computer interaction. For instance, *Spatial-Query-by-Sketch* (Egenhofer, 1997; Blaser and Egenhofer, 2000) enables its users to query spatial data by drawing a sketch map. *ASSIST* (Alvarado and Davis, 2001; Davis, 2002) interprets a mechanical sketch and predicts how the illustrated mechanism would behave. Similarly, *SketchIT* (Stahovich, 1997; Kurtoglu and Stahovich, 2002) interprets mechanical sketches and recreates new designs that realize the same functions. Landay and Myers (2001) developed a computer-aided GUI-design support systems, which interprets a hand-drawn GUI layout and generates a prototype program. While these systems were designed for specific tasks, the Electronic Cocktail Napkin (Gross 1994) was developed as an adaptive pen-based system for supporting the design of various diagrams, from flowcharts (Gross 1994) to architectural floor plans (Gross and Do, 2000). Similarly, *GeoRep* (Ferguson and Forbus, 2000) is a generic platform for sketch-based systems. *GeoRep* was applied to *sKEA* (Ferguson and Forbus, 2002; Forbus and Usher, 2002), which enables its user to teach the computer his or her knowledge by sketching a diagram.

Some systems combine a sketching interface with a speech interface. In *Sketch-and-Talk* (Egenhofer, 1996) a user queries spatial data by specifying a place of interest through a drawn sketch and voice annotations. Likewise, *QuickSet* (Cohen *et al.*, 1997; Johnston, 1998; Cohen *et al.*, 2000; Oviatt and

Cohen, 2000) is a multi-modal system for map-based tasks, which is operated by speech and pen input. Similarly, *nuSketch COA creator* (Forbus *et al.*, 2001; Ferguson and Forbus, 2002) is a multi-modal system for a map-based task (military operation planning) based on the *GeoRep* platform, and *ASSISTANCE* (Davis, 2002) facilitates mechanical design with sketch and verbal input.

The main focus of this paper is how such sketch-based systems deal with arrow symbols. Some systems accept only a single semantic role of arrow symbols. For instance, in Landay and Myers's (2001) computer-aided GUI-design system, arrow symbols are used only for specifying button-window relations—which window emerges or gets focus when a button is clicked. In *SketchIT*, arrow symbols specify the movable direction of mechanical components. In *ASSIST*, the user may use one arrow symbol for specifying the gravity direction. Such restrictions of arrow symbols to few semantic roles work effectively for specific tasks, since the ambiguity of arrow symbols is excluded. As a drawback, however, the users of these systems are forced to learn such restrictions of arrow symbols.

QuickSet and *nuSketch COA creator* accept arrow symbols with a variety of semantic roles. In *sKEA*, arrow symbols may represent arbitrary binary relations. These systems, however, have room for improvements, because the users have to specify the semantic role of every single arrow symbol, either by speech (e.g., in *QuickSet*), or the use of different shapes of arrow symbols (e.g., in *nuSketch COA creator*), by text input, or by menu selection (e.g., in *sKEA*).

Overall, most sketch-based systems limit a natural use of arrow symbols, due to the lack of a human-like ability to deduce the semantic roles of arrow symbols. One exception is *ASSISTANCE*, which automatically distinguishes the arrow symbols representing causality from those representing external force. This distinction, however, depends on domain-specific rules, which prevent a direct application to other sketch-based tasks.

4 Four Major Classes of Semantic Roles

Arrow symbols have a large variety of semantic roles. These semantic roles may be assigned to individual arrow symbols or to groups of arrow symbols (Kurata and Egenhofer, 2006b). To focus the discussion, this paper only considers the semantic roles assigned to individual arrow symbols, especially four major semantic roles that comprise the other detailed semantic roles. These four roles are determined in the following stepwise manner (Figure 1), based on the classification by Kurata and Egenhofer (2006). The completeness of these four major semantic roles is examined in Section 9.

First, we distinguished two groups of semantic roles depending on whether a single component or multiple components are required. Single-component arrow symbols are typically used for specifying the directional property of this component, using the arrow symbols' directionality; therefore, this semantic role is called *orientation*.

A basic characteristic of arrow symbols is to evoke the image of a transition. Such transitions may include spatial transitions (i.e., movement) as well as temporal transitions (i.e., continuous existence, possibly accompanied by a change). Thus, we consider the group of multi-component arrow symbols that represent transitions. The semantic roles of these arrow symbols are called *behavioral description*.

In the remaining cases, arrow symbols connect the components in a certain meaningful way without implying a transition. Due to their linearity, arrow symbols usually connect two elements (or two sets of elements). Two cases are distinguished depending on whether these components refer to different subjects or the same subject. Thus, we consider two additional semantic roles, called *association* and *annotation*. In an association, arrow symbols link two components, indicating the presence of an asymmetric relation between them, whereas in an annotation, arrow symbols attach a description (label) to another component.

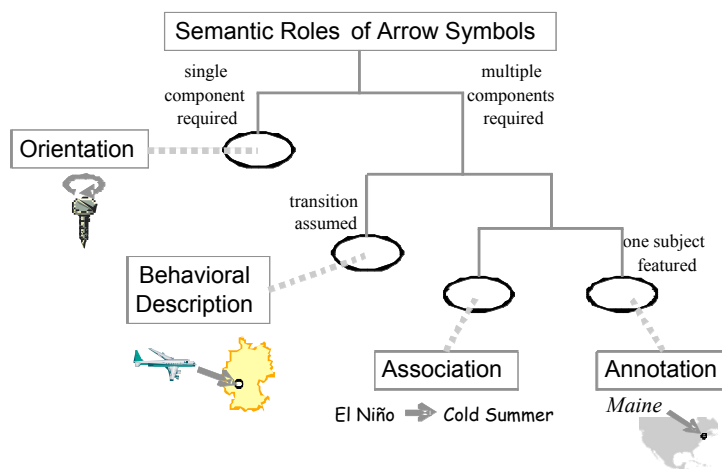


Figure 1. Deriving the four major semantic roles of arrow symbols, together with examples.

5 Individual Structures of Arrow Symbols

The components around each arrow symbol can be arranged differently, potentially influencing the arrow symbols' semantic roles. To enable a systematic analysis how the arrangement of components influences the semantic roles of arrow symbols, we develop a model of component arrangements around each arrow symbol, called the *individual structure*.

When an arrow symbol explicitly refers to a component, this component is located in front of, behind, or along the arrow symbols. As such, an arrow symbol is a deictic reference frame (Retz-Schmidt, 1988), which identifies three different conceptual areas where the components can be located (Figure 2). These three areas are called the *component slots* of an arrow symbol, or the *tail slot*, *body slot*, and *head slot*, respectively (Kurata and Egenhofer, 2005a). Every component, to which an arrow symbol explicitly refers, is assigned uniquely to one of the three component slots, thereby making the distinction of *tail components*, *body components*, and *head components*.

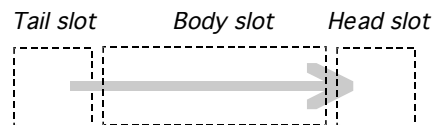


Figure 2. Three component slots associated with an arrow symbol.

The individual structure associated with an arrow symbol a (called a 's *individual structure*) is defined as a list of the components in a 's three component slots. It is denoted by a 3-tuple, $(C_{\text{tail}}(a), C_{\text{body}}(a), C_{\text{head}}(a))$, where $C_{\text{tail}}(a)$, $C_{\text{body}}(a)$, and $C_{\text{head}}(a)$ are the non-ordered sets of tail, body, and head components, respectively. For instance, the individual structure associated with the arrow symbol in Figure 3 is $(\{\text{a launching space shuttle (icon), "Launching", "Florida - 8:15am"}\}, -, \{\text{a space shuttle disposing booster (icon), "Booster Disposal", "8:17am"}\})$.

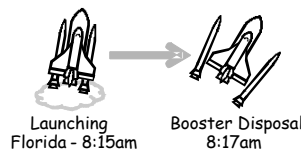


Figure 3. An arrow diagram that illustrate how the launching of a space shuttle in Florida at 8:15 am is followed by its booster disposal at 8:17am.

The components are categorized into *primary components (PCs)* and *modifier components (MCs)*. Primary components represent entities or concepts whose property, action, or relation is described by the arrow symbol. On the other hand, modifier components characterize such primary components or their property, action, or relation. As such, primary components are fundamental for establishing the semantics of arrow symbols, while modifier components are optional. For instance, in Figure 3, two icons are primary components that represent "a launching space shuttle" and "a space shuttle disposing booster," while the labels "Launching," "Florida - 8:15am," "Booster Disposal," and "8:17am" are modifier components, which characterize these icons. The

distinction of primary and modifier components is purely conceptual. There are, however, the following conventions and rules that are useful for the distinction of primary and modifier components in a visual domain:

- icons are usually primary components;
- text labels attached to icons are usually modifier components;
- if an arrow symbol refers to only one component through three slots, this component is a primary component, because this component is the subject in *orientation* (Section 6); and
- if the head slot has only one component, this component is a primary component, because the modifier component cannot be used in the head slot (Section 7).

The primary components are further categorized into the following four subclasses:

- A *location* (PC_L) is a position in space (e.g., “*Boston*” in Figure 4a).
- A *moment* (PC_M) is a position in time (e.g., “*9:00pm*” and “*11:00pm*” in Figure 4b).
- An *object* (PC_O) is an entity or its unit, which exists in a physical or conceptual space, and takes an action (e.g., a vehicle in Figure 4a) or interacts with another object. Objects are *continuants*, which endure through some extended interval of time (Worboys and Hornsby, 2004).
- An *event* (PC_E) occurs in time, either at an instant or over an interval (e.g., a “*Meeting*” in Figure 4b). It is characterized by a set of changes that the event triggers. Events are *occurents*, which happen and are then gone (Worboys and Hornsby, 2004).

The modifier components may also include location-like components (e.g., Florida in Figure 3) and moment-like components (e.g., “8:15am” in Figure 3), but they are not considered locations and moments, because they are optional, not fundamental, for establishing the semantics by the arrow symbols.



Figure 4. Two arrow diagrams with (a) a location (“*Boston*”) and (b) two moments (“*9:00am*” and “*11:00am*”).

With the distinction of primary and modifier components (PC and MC) and further distinction of primary components’ subclasses (PC_L , PC_M , PC_O , and PC_E), the pattern of the individual structure is defined as follows:

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pattern_of_individual_structure ::= "(" tail_components
    "," body_components "," head_components ")"
tail_components ::= [components]
body_components ::= [components]
head_components ::= [components]
components ::= component [components]
component ::= PCL | PCM | PCO | PCE | MC

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For instance, the individual structures of arrow symbols in Figures 3, 4a, and 4b have the patterns of $(MC\ PC_O, MC, PC_E\ MC)$, $(PC_O, -, PC_L)$, and (PC_M, PC_E, PC_M) , respectively.

Kurata and Egenhofer (2005a; 2006b) categorized the components of arrow symbols into five types—*objects*, *moments*, *locations*, *modifiers*, and *notes*. In this categorization, notes are not clearly defined at the semantic level, because locations and moments (in their terminology) sometimes play a similar role as notes (i.e., characterizing the other element). To overcome such confusion we introduce the distinction of primary components and modifier components, such that modifier components include all components that characterize the other elements.

6 Formats of Individual Structures

To use an arrow symbol, one has to arrange the components around the arrow symbol following certain rules. These rules determine the format of the individual structure. This section identifies the sets of such formats for the four classes of semantic roles.

Orientation

An arrow symbol for *orientation* is attached to one component (*subject*) and specifies a certain directional property of the subject. The arrow symbol points to, originates from, or passes through or by the subject, typically implying that the directional property is related to an outgoing action, a passing action, or an incoming action (Figure 5). Accordingly, *orientation* corresponds to the three formats in Figure 6.

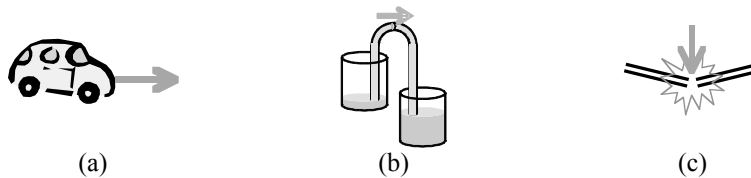
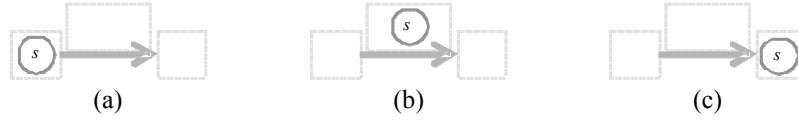


Figure 5. Diagrams with an arrow symbol for *orientation*, specifying (a) a vehicle's moving direction, (b) a direction of a liquid flow at a curving point of a tube, and (c) the direction of an external force by which a board cracks.

Figure 6. Formats of individual structures for *orientation* (*s*: subject).

Obviously, the subject must be a primary component, which represents an independent entity or concept. Among the four subclasses of primary components, a moment cannot be the subject, since the moment is a zero-dimensional concept and may have no directional property. On the other hand, the arrow diagrams in Figure 5, whose subjects are a vehicle, a place, and a board-cracking event, indicate that the subject may be an object, a location, and an event, respectively. Consequently, when an arrow symbol is used for *orientation*, its individual structure satisfies one of the basic patterns in Table 1. In these basic patterns, $PC_{L|O|E}$ represents PC_L , PC_O , or PC_E , and each blank in the parenthesis indicates that the corresponding component slot may be empty or filled by optional components. For instance, the pattern $(PC_L \quad MC, MC, -)$ satisfies the basic pattern $(PC_{L|O|E}, \quad , \quad)$.

Table 1. Basic patterns of individual structures for *orientation* (*s*: subject).

Format	Basic patterns
(s, \quad , \quad)	$(PC_{L O E}, \quad , \quad)$
(\quad , s, \quad)	$(\quad , PC_{L O E}, \quad)$
(\quad , \quad , s)	$(\quad , \quad , PC_{L O E})$

Behavioral description

An arrow symbol for *behavioral description* illustrates the transition of a component (*subject*). In addition to the subject, the arrow symbol may refer to the components that specify spatial or temporal positions. These components are called *transition-related position specifiers*. If located in the tail, body, and head slots, they specify the origin, intermediate place, and destination of the transition (Figure 4a), or the start time, intermediate time, and end time of the transition (Figure 4b), respectively. The arrow symbol for *behavioral description* may also refer to the components with which the subject interacts during the transition (Figures 7a-b). These components are called the *participants*. If the participants are located in the tail, body, and head slots, the interaction takes place before, during, and after the transition, respectively.

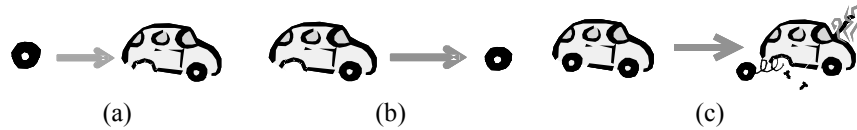


Figure 7. Diagrams with an arrow symbol for *behavioral description*, illustrating (a) installation of a wheel, (b) removal of a wheel, and (c) change of a vehicle. In (a-b) the wheel is the subject while the vehicle is the participant, while in (c) the vehicle, which is illustrated in both tail and head slots, is the subject.

When an arrow symbol is used for a *behavioral description*, its individual structure must satisfy the following constraints:

- The subject s is located in any component slot (Figures 7a-b), except when the diagram highlights the change of the subject (Figure 7c).
- In addition to the subject s , the arrow symbol refers to at least one transition-related position specifier p_o or one participant p_a ; otherwise the arrow symbol refers to the subject alone and, accordingly, the semantic role of this arrow symbol becomes *orientation* instead of *behavioral description*.
- The participant p_a cannot be located at the same place with the subject s ; otherwise the diagram no longer implies the interaction between s and p_a . In addition, we assume that multiple participants cannot be located at the same place (if apparently multiple participants are located at the same place, they are considered a single participant as a group). Accordingly, p_a cannot be located in a tail or head slot that already contains s or another p_a .
- The transition-related position specifier p_o cannot be located at the same location with the subject s , the participant p_a , or another transition-related position specifier; otherwise, p_o is regarded an *adjectival component* (Section 7). Accordingly, p cannot be located in a tail or head slot that already contains s , p_a , or another p_o .
- The body slot may contain the subject s , one or more p_a , and one or more p_o at the same time, because it has length.

These constraints determine the formats of individual structures for *behavioral description* (Table 2).

Table 2. Formats of individual structure for *behavioral description* (s : subject, p_a : participant, p_o : transition-related position specifier). $[X]^n$ represents one or more X .

		Formats		
Slots that contain the subject	Head			
	Tail			
	Body			
	Head & Tail			

The subject s , each participant p_a , and the transition-related position specifier p_o must be a primary component, as they exist independently. Among the four subclasses of primary components, either an object or an event can be the subject s , since objects and events may change their spatial or temporal position specifiers, but locations and moments cannot. Each participant p_a must be an object, an event, or a location, because they can be a target of action, but a moment cannot. Each transition-related position specifier p_o is obviously either a location or a moment. Consequently, when an arrow symbol is used for *behavioral description*, its individual structure must satisfy one of the basic patterns in Table 3.

Table 3. Basic patterns of individual structures for *behavioral description* (s : subject, p_a : participant, p_o : transition-related position specifier).

	Format	Basic patterns	
Slots that contain the subject	Head	$(s, , p_a p_o)$	$(PC_{O E}, , PC_{L M O E})$
		$(s, [p_a p_o]^n,)$	$(PC_{O E}, [PC_{L M O E}]^n,)$
		$(s, [p_a p_o]^n, p_a p_o)$	$(PC_{O E}, [PC_{L M O E}]^n, PC_{L M O E})$
	Body	$(p_a p_o, , s)$	$(PC_{L M O E}, , PC_{O E})$
		$(, [p_a p_o]^n, s)$	$(, [PC_{L M O E}]^n, PC_{O E})$
		$(p_a p_o, [p_a p_o]^n, s)$	$(PC_{L M O E} [PC_{L M O E}]^n, PC_{O E})$
	Tail	$(p_a p_o, s,)$	$(PC_{L M O E}, PC_{O E},)$
		$(, s, p_a p_o)$	$(, PC_{O E}, PC_{L M O E})$
		$(p_a p_o, s, p_a p_o)$	$(PC_{L M O E}, PC_{O E}, PC_{L M O E})$
		$(p_a p_o, s[p_a p_o]^n,)$	$(PC_{L M O E}, PC_{O E}[PC_{L M O E}]^n,)$
		$(, s[p_a p_o]^n, p_a p_o)$	$(, PC_{O E}[PC_{L M O E}]^n, PC_{L M O E})$
		$(p_a p_o, s[p_a p_o]^n, p_a p_o)$	$(PC_{L M O E}, PC_{O E}[PC_{L M O E}]^n, PC_{L M O E})$
Head & Tail	$(s, , s)$	$(PC_{O E}, , PC_{O E})$	
	$(s, [p_a p_o]^n, s)$	$(PC_{O E}, [PC_{L M O E}]^n, PC_{O E})$	

Annotation

An arrow symbol for *annotation* attaches a text component (*label*) to another component (*subject*), thereby specifying the subject’s name, category, status, spatial position, temporal position, and so on (Figure 8). Conventionally, *annotation* corresponds to only one format (Figure 9), where an arrow symbol originates from the label and points to the subject. This format implies that the label is assigned to the subject.

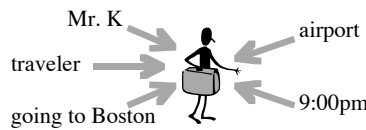


Figure 8. An arrow diagram with five arrow symbols, all used for *annotation*.



Figure 9. The unique format of individual structures for *annotation* (*l*: label, *s*: subject).

The label that describes the subject is naturally a modifier component, while the subject is a primary component of any subcategory. Consequently, when an arrow symbol is used for *annotation*, its individual structure must satisfy the basic pattern in Table 4.

Table 4. The basic pattern of individual structures for *annotation* (*l*: label, *s*: subject).

Format	Basic pattern
(l, s)	$(MC, PC_{L MO E})$

Association

An arrow symbol for *association* links two components (*subjects*), indicating the presence of an asymmetric relation between them. Conventionally, these two subjects are placed in the tail and head slots (Figure 10), such that these two subjects look equally emphasized while their order is highlighted.

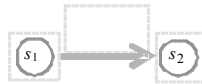


Figure 10. The unique format of individual structures for *association* (s_1, s_2 : associated subjects)

The asymmetric relation that holds between the two subjects is called the *effective relation*. The caption, legend, or *adverbial component* (Section 7) may mention the effective relation; otherwise, the diagram reader has to infer the effective relation from the context or their own knowledge about typical relations between the subjects. The effective relation may provide an ordering rationale, which determines the order of the associated subjects (Table 5).

Each subject must be a primary component, which represents an independent entity or concept. Any subcategory of primary components can be the subject, as long as people can identify an effective relation between the subjects. Accordingly, when an arrow symbol is used for *association*, its individual structure must satisfy the basic pattern in Table 6.

Table 5. Examples of *association* by arrow symbols, where the effective relations between the subjects may naturally determine the subjects' order.

Example	Associated Subjects	Effective Relation	Ordering Rationale
El Niño→Fish Catch_	El Niño, Fish Catch _	Causality	logical order
Plan→Do→See	Plan, Do, See	work process	temporal order
Niagara Falls→Lake Ontario	Niagara Falls, Lake Ontario	water flow	spatial order (high to low)
Paris→Eiffel Tower Eiffel Tower→Paris	Paris, Eiffel Tower	a city and its landmark	imaging order

Table 6. The basic pattern of individual structures for *association* (s_1, s_2 : associated subjects).

Format	Basic patterns
(s_1, s_2)	$(PC_{L MOE}, PC_{L MOE})$

7 Optional Components

Individual structures of arrow symbols may have optional components, which enrich the diagram's semantics. Two types of optional components are distinguished: *adjectival components* and *adverbial components*. An adjectival component describes a character of its nearby component, such as name, spatial position, and temporal position (Figure 11). On the other hand, an adverbial component provides the following information related to the semantic role of the arrow symbol:

- When an arrow symbol is used for *orientation*, its adverbial component describes the type, name, or scale of the directional property (Table 7a₁-a₃).
- When an arrow symbol is used for *behavioral description*, its adverbial component describes the transition's type, scale, or where or when the transition takes place (Table 7b₁-b₃).
- When an arrow symbol is used for *association*, its adverbial component describes the effective relation that associates the components, or where or when the relation holds (Table 7c₁-c₃).

Adjectival and adverbial components do not have to be represented by labels of adjectives and adverbs, respectively. For instance, in Figure 11, two labels of nouns represent adverbial components. The names of adjectival and adverbial

components emphasize that these components are functionally analogous to the adjectives and adverbs in languages. Naturally, both adjectival and adverbial components are modifier components. An adjectival component is placed in the same slot with the primary component that it modifies, while an adverbial component is placed in the body slot, normally around its center, implying that the adverbial component is assigned to the entire arrow symbol.

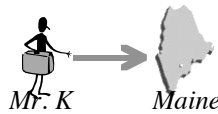


Figure 11. An arrow diagram with two adjectival components, “Mr. K” and “Maine,” which specify the name of their nearby components.

Table 7. Information provided by adverbial components.

		Provided Information		
Semantic Role	Orientation	external force → <input type="checkbox"/> (a ₁) property type	f_1 → <input type="checkbox"/> (a ₂) property name	50 kgm/s ² → <input type="checkbox"/> (a ₃) property scale
	Behavioral Description	● install → (b ₁) transition type	10 mph → (b ₂) transition scale	 (b ₃) where or when the transition takes place
	Association	belong to → (c ₁) effective relation	2006 Germany W-cup → tourist ? (c ₂) where or when the relation holds	

8 Deriving Interpretations from Individual Structures

Let us assume that we already know the individual structure of an arrow symbol *a*. By combining the formats of individual structures (Section 6) and the rules of optional components (Section 7) we can parse *a*'s individual structure and determine all of *a*'s potential semantic roles among *orientation*, *behavioral description*, *annotation*, and *association*.

Let a 's individual structure be $s_{\text{ind}}(C_{\text{tail}_i}, C_{\text{body}_{ij}}, C_{\text{head}_{ij}})$, where $C_{\text{tail}}(a)$, $C_{\text{tail}}(a)$, $C_{\text{body}}(a)$, and $C_{\text{head}}(a)$ represent the sets of tail, body, and head components, respectively. Let σ_1 - σ_4 be *orientation*, *behavioral description*, *annotation*, and *association*, respectively. The arrow symbol a may have a semantic role σ_i only if $s_{\text{ind}}(a)$ satisfies one of σ_i 's formats and every extra component in $s_{\text{ind}}(a)$ is considered as either an adverbial component or an adjectival component. Let $(\overline{T}_{ij}, \overline{B}_{ij}, \overline{H}_{ij})$ be the j^{th} basic pattern of the semantic role σ_i (Tables 1, 3, 4, and 6). Assume we have a function $ct(c)$ that gives the type of a component c . With this setting, the *Arrow-Semantics Interpreter* algorithm deduces all of a 's potential semantic roles as follows:

```

1: Result  $\leftarrow$  {}
2:  $(T, B, H) \leftarrow (\{ct(c) | c \in C_{\text{tail}}(a)\}, \{ct(c) | c \in C_{\text{body}}(a)\}, \{ct(c) | c \in C_{\text{head}}(a)\})$ 
3: For every semantic role class  $\sigma_i$ 
4:   For  $\sigma_i$ 's every basic pattern  $(\overline{T}_{ij}, \overline{B}_{ij}, \overline{H}_{ij})$ 
5:     If  $(\overline{T}_{ij} \subset T) \wedge (\overline{B}_{ij} \subset B) \wedge (\overline{H}_{ij} \subset H)$  then
6:        $(T^*, B^*, H^+) \leftarrow (T \setminus \overline{T}_{ij}, B \setminus \overline{B}_{ij}, H \setminus \overline{H}_{ij})$ 
7:        $(T^*, B^*, H^*) \leftarrow (T^*, B^*, H^+)$ 
8:       IF  $\sigma_i \neq$  "Annotation" then remove all MC from  $B^*$ 
9:        $S \leftarrow T^* \cup B^* \cup H^*$ 
10:      If  $(\{PC_M, PC_O, PC_E, PC_L\} \cap S = \emptyset)$  and  $\_$ 
           $\neg((T^* \neq \emptyset) \wedge (\overline{T}_{ij} = \emptyset)) \vee ((B^* \neq \emptyset) \wedge (\overline{B}_{ij} = \emptyset)) \vee$ 
           $((H^* \neq \emptyset) \wedge (\overline{H}_{ij} = \emptyset))$   $\_$ 
          then add  $\sigma_i$  to Result
11:     EndIf
12:   Next
13: Next
14: End

```

The triple (T, B, H) shows the pattern of a 's individual structure $s_{\text{ind}}(a)$, while (T^*, B^*, H^+) shows the extra components in $s_{\text{ind}}(a)$ under the assumption that (T, B, H) satisfies the basic pattern $(\overline{T}_{ij}, \overline{B}_{ij}, \overline{H}_{ij})$. If this assumption is correct, every component in (T^*, B^*, H^+) must be either an adverbial component or an adjectival component and, therefore, every component in (T^*, B^*, H^*) must be an adjectival component after line 8. Since each adjectival component must be a modifier component, T^* , B^* , and H^* cannot include a primary component PC_L , PC_M , PC_O , or PC_E (line 10). In addition, if T^* , B^* , and H^* contain an adjectival component, there must be at least one component in \overline{T}_{ij} , \overline{B}_{ij} , and \overline{H}_{ij}

that the adjectival component modifies (line 10). If these two conditions are satisfied, the semantic role σ_i is recorded as a 's potential semantic role. After checking all basic patterns of the four classes of semantic roles, the *Result* captures all of a 's potential semantic roles.

9 Evaluation

To evaluate the ASI, we developed a software prototype, which deduces all potential semantic roles of an arrow symbol from the pattern of its individual structure (Figure 12). At this stage, the user has to specify the individual structure's pattern manually. The automation of this process is a subject for future work.

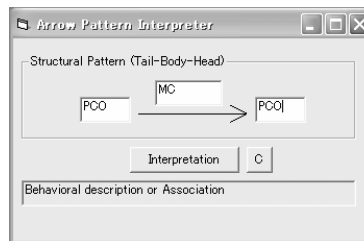


Figure 12. A screenshot of the ASI's prototype.

Sample arrow symbols for the assessment were collected from the introductory GIS textbook, *Geographical Information Systems and Computer Cartography* (Jones, 1997), because this material satisfies the following conditions:

- the material contains a sufficient number of figures that contain arrow symbols;
- the material is supposed to be read by people without special education or training in diagram reading;
- the figures are neither drawn by few designers nor do they adhere to in-house aesthetic standards (this is why newspapers and magazines are avoided); and
- the semantic roles of sample arrow symbols are not biased (Figure 14a).

We also examined two other textbooks in biology and astronomy, but the semantic roles of arrow symbols in these two textbooks are considerably biased. The biology textbook predominantly uses arrow symbols for illustrating chemical reactions and organisms' movements (which both belong to *behavioral descriptions*), while the astronomy textbook frequently uses arrow symbols for illustrating an interval, which is not captured by the four classes of semantic roles.

The correct semantic roles of sample arrow symbols were assigned manually based on the figures plus context, sometimes drawn from the caption and the body text. These assigned semantic roles were confirmed with a vote by a pool of 14 human subjects (Kurata, 2007).

From the GIS textbook, 304 arrow symbols in 64 figures were collected. Some figures contain a large number of similar arrow symbols (Figure 13). Since these similar arrow symbols, if counted individually, may distort the statistical result, one representative arrow symbol was selected for every set of similar arrow symbols in the same figure (i.e., arrow symbols with the same semantic role and the same pattern of individual structure). This selection reduced the sample set to 94 arrow symbols, which were used for the assessment. This selection process did not bias the proportion of the arrow symbols that belong to the four classes of semantic roles (Figure 14).

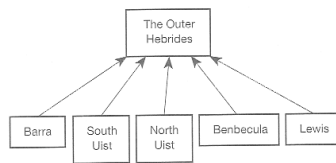


Figure 13. A figure from Jones (1997) that contains multiple similar arrow symbols.

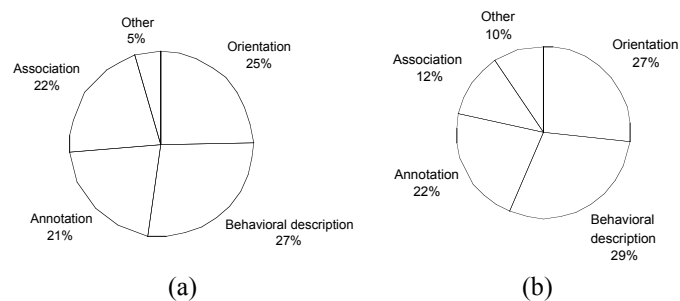


Figure 14. Semantic roles of (a) 304 arrow symbols found in the textbook and (b) 94 arrow symbols selected from the 304 arrow symbols for the assessment.

The evaluation started with counting the number of potential semantic roles that the ASI deduced for each sample arrow symbol. Then, we counted the number of sample arrow symbols whose interpretation yields:

- *exact match*, where the ASI deduced only one potential semantic role and it matches the correct semantic role;
- *partial match*, where the ASI deduced multiple potential semantic roles, one of which matches the correct semantic role;

- *oversight*, where the potential semantic roles deduced by the ASI do not include the correct semantic role, even though the correct semantic role is either *orientation*, *behavioral description*, *annotation*, or *association*; and
- *no-answer*, where the potential semantic roles deduced by the ASI do not include the correct semantic role, because the correct semantic role is not among the four classes of semantic roles.

Finally, we calculated the detection rate (i.e., the proportion of sample arrow symbols for which the ASI deduced potential semantic roles that includes the correct semantic role) and compared it with the detection rate under random choices.

Result

In most cases the ASI deduced one or two potential semantic roles (Figure 15), which indicates that the ASI reduces the ambiguity of arrow symbols, since there are initially four choices. For 6% of the sample arrow symbols, however, the ASI concluded that the semantic role of the given arrow symbol is not found in our four semantic roles. On average, the ASI deduced 1.31 potential semantic roles per arrow symbol.

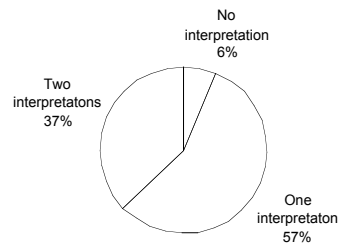


Figure 15. The number of semantic roles that the ASI deduced for the 94 arrow symbols.

For $44\% + 35\% = 79\%$ of the sample arrow symbols, the ASI successfully detected the correct semantic role (Figure 16). Among these successful cases, 44% of the sample arrow symbols yielded an exact match, which requires no further processing narrow down the potential semantic roles. On the other hand, for $11\% + 10\% = 21\%$ of the sample arrow symbols, the ASI failed to detect the correct semantic role. Among these unsuccessful cases, 10% of the sample arrow symbols were unable to be interpreted because their correct semantic roles were unsupported by the current ASI.

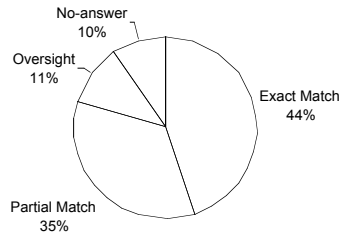


Figure 16. The proportion of the four types of interpretation results for the 94 arrow symbols.

Assume a computer randomly selects zero, one, or two semantic roles from the four classes at the probability of 6%, 57%, and 37%, respectively (Figure 15). Then, the *expected detection rate* (the probability that the randomly-selected potential semantic roles include the correct one) is 30% with respect to the 94 sample arrow symbols. On the other hand, the ASI's detection rate is 79%, which is much higher than the expected detection rate under random selection. This result indicates the robustness of the ASI's interpretation.

Analysis of Misinterpretations

To identify the reasons for the misinterpretations so that the ASI could be improved, we analyzed the sample arrow symbols whose interpretation yielded *oversight*, *no-answer*, or *partial match*.

Oversight: Eleven Cases

In our samples, *oversight* occurred when a component is omitted (Figure 17a) or an unexpected format is used (Figure 17b). The omission of components occurs when they are obvious from the caption, legend, body text, or context. Accordingly, the use of these information sources is a key for the solution.

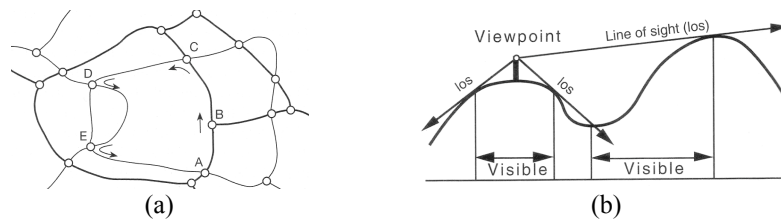


Figure 17. Figures from Jones (1997) where arrow symbols yield *oversight*, because (a) the subject of *behavioral description* is omitted and (b) the arrow symbols for *orientation* refer to two locations instead of one.

No-Answer: Nine cases

Through the analysis of nine *no-answer* cases, the following semantic roles of arrow symbols are newly identified:

- to illustrate an interval (Figure 18a);
- to highlight a certain point in the space (Figure 18b); and
- to imply a gradation (Figure 18c)

The first semantic role is normally assigned to a pair of arrow symbols, instead of individual arrow symbols. Such group-oriented semantic roles of arrow symbols are related to the formations of the arrow symbols (Kurata and Egenhofer, 2006b). The second semantic role can be regarded as a special case of *annotation* where the label is omitted, because its message is trivial (for instance, *look here*). The third semantic role seems to be a less frequent case.

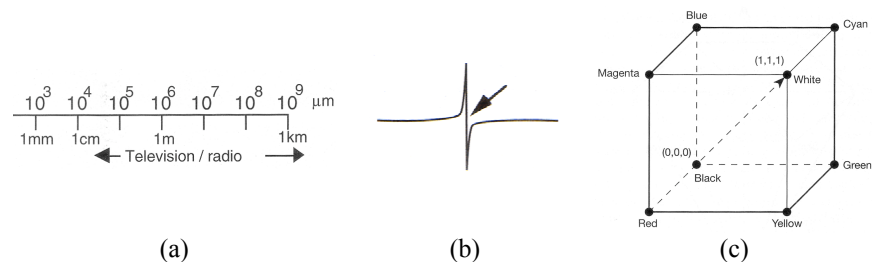


Figure 18. Figures from Jones (1997) where arrow symbols yield *no-answer*, because they are used for (a) illustrating an interval, (b) highlighting, and (c) implying a gradation, instead of the four classes of semantic roles.

Partial Match: Thirty-three Cases

Table 8 summarizes the patterns of individual structures of 33 arrow symbols that have yielded a *partial match*. All *partial match* cases originate from the overlap of the patterns that correspond to *behavioral description* and the patterns that correspond to *association*. Among such patterns, $(PC_o[MC]^*, [MC]^*, PC_o[MC]^*)$ is frequently observed (24 out of 33 cases). This pattern may be used for three scenarios:

- to illustrate an interaction between a subject and a participant;
- to illustrate the change of a subject; and
- to associate two different subjects.

The first two belong to *behavioral description*, while the last one belongs to *association*. The following clues may be useful for the distinction of these three scenarios:

- If the arrow symbol has non-simple shape, an arrow symbol is usually used for *behavioral description* (in this case, the first scenario), because the arrow symbol's shape often implies the course of spatial transition.

- If two PC_O s cannot be considered two different states of the same subject, an arrow symbol may not illustrate a change.
- If two PC_O s are both immovable, an arrow symbol may not illustrate an interaction.

This implies that an arrow's visual information and background knowledge about its components' characteristics (mobility and possible states) may contain additional information critical to the precise interpretation of the diagram's semantics. Similar ideas can be applied for the distinction of *behavioral description* and *association* in other cases.

Table 8. Patterns of the individual structures of the 33 arrow symbols that have yielded a *partial match*.

Pattern of Individual Structure	Deduced Semantic Roles		Num. of Samples
	Correct	Unnecessary	
$(PC_O[MC]^*, [MC]^*, PC_O[MC]^*)$			17
$(PC_O[MC]^*, [MC]^*, PC_E[MC]^*)$	<i>behavioral</i>	<i>association</i>	3
$(PC_O[MC]^*, [MC]^*, PC_O[MC]^*)$	<i>description</i>		3
$(PC_E[MC]^*, [MC]^*, PC_L[MC]^*)$			2
$(PC_O[MC]^*, [MC]^*, PC_O[MC]^*)$	<i>association</i>	<i>behavioral</i>	7
$(PC_O[MC]^*, [MC]^*, PC_L[MC]^*)$		<i>description</i>	1

10 Conclusions and Future Work

This paper developed an algorithm for deducing potential semantic roles of arrow symbols. The Arrow-Semantics Interpreter makes use of the requirements on the arrangement of components, which are specific to each class of semantic roles. The assessment of the algorithm's inferences showed that it deduces on average 1.31 potential semantic roles, which include in 79% the correct answer. This result indicates that the pattern of the components' arrangements around each arrow symbol is critical for deriving the interpretation of the arrow symbol. The analysis of misinterpreted samples revealed that the ASI will be improved by addressing explicitly to the following issues:

- detection of omitted components based on captions, legends, and context;
- deduction of semantic roles assigned to a group of arrow symbols;
- judgment of valid semantic roles, making use of arrow's visual information and knowledge about the components' mobility and possible states; and
- support of additional semantic roles and formats, if necessary.

In the future, we want to apply the ASI to sketch-based systems in a practical way, such that their users can draw arrow symbols freely and naturally in their

sketches. To achieve this goal, we further need the following techniques, which are left for future research:

- Symbol and text recognition techniques to detect arrow symbols and components in raw sketches.
- Automated linking between arrow symbols and components (i.e., judging whether each arrow symbol refers to each component and, if so, in which slot).
- Automated distinction of component types. Some diagrammatic conventions help to make the distinction between primary and modifier components (Section 5), but further distinction of their subclasses requires knowledge about the components' characteristics.

Another challenge is to furnish details to the interpretations, since they might be too coarse for certain applications.

Arrow symbols are frequently used in people's everyday communication. Especially, they are essential for the communication of spatial and spatio-temporal information. We, therefore, believe that deeper understanding of arrow symbols is important for enriching the interaction between people and machines.

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We would like to appreciate Kate Beard, Mike Worboys, Kathleen Stewart Hornsby, Werner Kuhn, and Ronald Ferguson for their advice. This work was partially supported by the National Geospatial-Intelligence Agency under grant numbers NMA201-01-1-2003 and NMA401-02-1-2009.

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