Introductory Methods for Area Data

Bailey and Gatrell Chapter 7

Lecture 19

November 17, 2009

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Generalized Least Squares

Ordinary least squares does not cover the possibility of second order effects so we need to resort to generalized least squares

$$Y = X\beta + U$$

U is a zero mean vector of errors with variance – covariance matrix C

$$E(U) = 0$$

$$E(UU^T) = C$$

Generalized least squares provides a way to model both first and second order effects but we do not initially know C

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Tests for Spatial Autocorrelation

Correlogram provides a method to assess second order effects.

We want to know how significant second order effects are

Test significance of departure from the hypothesis of zero spatial correlation

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Tests for Spatial Autocorrelation

Are spatial autocorrelation measures significantly larger than one would expect to arise by chance from a spatially independent process.

Null hypothesis of zero correlation

$\mathbf{H_0}: \rho = \mathbf{0}$

$\mathbf{H_1}: \rho \neq \mathbf{0}$

Two approaches for testing for departure from this hypothesis

- Random permutation test
- Approximate sampling distribution

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Tests for Spatial Autocorrelation

- Random permutation test

n values associated with the areas A_i are randomly permutated

Compute Moran's I for each permutation to build up an empirical distribution of I

Compare the value of I for the observed arrangement of y_i to this empirical distribution of I – is the observed value an extreme value in this distribution?

There are n! possible permutations – Monte Carlo approach allows a random sampling from the n! possible permutations.

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Tests for Spatial Autocorrelation

Normalization assumption

Values are assumed to be the result of n independent drawings from a normal population. The observed map is one realization of the underlying normal distribution. Then I has a sampling distribution that is approximately normal

$$E(I) = -\frac{1}{(n-1)}$$

$$VAR(I) = \frac{n^2(n-1)S_1 - n(n-1)S_2 - 2S_0^2}{(n+1)(n-1)^2 S_0^2} \qquad S_0 = \sum\sum_{i \neq j} w_{ij}$$

$$S_1 = \frac{1}{2}\sum\sum_{i \neq j}(w_{ij} + w_{ji})^2 \qquad S_2 = \sum_k \left(\sum_j w_{kj} + \sum_i w_{ik})^2 \right)$$

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Tests for Spatial Autocorrelation

Qualifications

Potential problem if **I** has been calculated from residuals of a regression model

Residuals are subject to p linear constraints due to p parameters estimated in regression model – so residuals are automatically correlated to some extent

If p << n then usually OK

Strictly should use adjustments to mean and variance of approximate sampling distribution of **I**

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Generalized Least Squares

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{U}$$

Y is a (n x 1) vector of random variables

X is a matrix of (n x p) explanatory variables

β is a (p x 1) vector of coefficients

U is a (n x 1) vector of zero mean random variables with variance-covariance matrix **C**

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{C}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{C}^{-1}y$$

$$VAR(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}^T\mathbf{C}^{-1}\mathbf{X})^{-1}$$

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Spatial Regression Models

Instead of trying to specify ***C*** directly, it is modeled indirectly through interaction schemes

Spatial regression models include relationships between variables and their neighboring values

Can involve additional parameters to be estimated

ρ is a parameter to be estimated that determines the direction and magnitude of the spatial neighborhood effect.

w_{ij} are weights that determine the relative influence of location j on location i

Indirectly this interaction specifies a form of $\mathbf{C}$

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Spatial Regression Models

Autoregressive models

Originated in time series analysis $Z_t = \rho Z_{t-1} + \varepsilon_t$

Various types of spatial autoregressive models are employed

- Simultaneous autoregressive models - **SAR**
- Conditional autoregressive models - **CAR**
- Moving Average model - **MA**

These models differ in the specification of the inverse covariance functions

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Pure Autoregressive Model - SAR

The explanatory variable consists of only a spatial lag for the dependent variable – a weighted average of neighboring values

$$\mathbf{Y} = \rho \mathbf{W}\mathbf{Y} + \varepsilon$$

ρ is the spatial autoregressive coefficient

$\mathbf{W}$ is the spatial weight matrix for y

ε is the the vector of zero mean constant variance errors

For individual observations: $y_i = \rho(\sum_j w_{ij} y_j) + \varepsilon_i$

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Pure Autoregressive Model

A higher order pure autoregressive model

$$\mathbf{Y} = \rho_1 \mathbf{W}^{(1)}\mathbf{Y} + \rho_2 \mathbf{W}^{(2)}\mathbf{Y} + \cdots \rho_s \mathbf{W}^{(s)}\mathbf{Y} + \varepsilon$$

Where the $W^{(1)}$, $W^{(2)}$, $W^{(3)}$ are the weights matrices corresponding to each order of neighborhood and the ρ_1, ρ_2, ρ_3 are the matching autoregressive coefficients

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Mixed Regressive Spatial Autoregressive Model

$$\mathbf{Y} = \mathbf{X\beta} + \rho\mathbf{WY} + \mathbf{U}$$

Includes explanatory variables in addition to the spatial lag terms

X is a vector of explanatory variables

β are the coefficients of the explanatory variables

Expresses how **Y** relates to its values in surrounding locations while controlling for the influence of other explanatory variables

Spatial Regression Models

Another version of a SAR model is the autocorrelated errors model.

$$\mathbf{Y} = \mathbf{X\beta} + \mathbf{U}$$
$$\mathbf{U} = \rho\mathbf{WU} + \varepsilon$$

Spatial dependence in the error term is specified as a spatial process

This model is applied when there appears to be significant spatial autocorrelation, but tests for spatial lag effects do not suggests that inclusion of the latter would provide a significant improvement.

Spatial Error Models

Autocorrelated errors model

Observations interdependent through unmeasured variables that are correlated across space or measurement error that is correlated with space

- A nuisance that arises because we can not model all the facets of a geographical region that may influence all nearby locations
- May also arise from boundaries that are not perfect measures

Theoretically possible to eliminate this type of spatial dependence with proper explanatory variables and correct boundaries of observations

Spatial Error Models

$$\begin{aligned}\mathbf{Y} &= \mathbf{X\beta} + \rho\mathbf{WU} + \varepsilon \\ &= \mathbf{X\beta} + \rho\mathbf{W}(\mathbf{Y} - \mathbf{X\beta}) + \varepsilon \\ &= \mathbf{X\beta} + \rho\mathbf{WY} - \rho\mathbf{WX\beta} + \varepsilon\end{aligned}$$

General trend component

Neighboring y values

Neighboring trend values

Spatial Regression Models

Derivation of C for SAR $\mathbf{U} = \rho \mathbf{W} \mathbf{U} + \varepsilon$

$$\mathbf{C} = E(\mathbf{U}\mathbf{U}^T)$$
$$= E(\mathbf{I} - \rho\mathbf{W})^{-1}\varepsilon\varepsilon^T((\mathbf{I} - \rho\mathbf{W})^{-1})^T)$$
$$= (\mathbf{I} - \rho\mathbf{W})^{-1}E(\varepsilon\varepsilon^T)((\mathbf{I} - \rho\mathbf{W})^{-1})^T)$$
$$= (\mathbf{I} - \rho\mathbf{W})^{-1}\sigma^2\mathbf{I}((\mathbf{I} - \rho\mathbf{W})^{-1})^T)$$
$$= \sigma^2(\mathbf{I} - \rho\mathbf{W})^{-1}((\mathbf{I} - \rho\mathbf{W})^T)^{-1})$$
$$= \sigma^2(\mathbf{I} - \rho\mathbf{W})^T((\mathbf{I} - \rho\mathbf{W}))^{-1}$$

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Spatial Regression Models

$$\mathbf{C} = \sigma^2(\mathbf{I} - \rho\mathbf{W})^T((\mathbf{I} - \rho\mathbf{W}))^{-1}$$

To obtain a valid covariance matrix $(\mathbf{I} - \rho\mathbf{W})$ must be invertable

To ensure invertability:

Restrictions apply to ρ

W is standardized to have row values of 1 which constrains ρ to

$$-1 \le \rho \le 1$$

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Spatial Regression Models

The form of the covariance is determined by the spatial process

For SAR model

$$\mathbf{C} = \sigma^2(\mathbf{I} - \rho\mathbf{W})^T((\mathbf{I} - \rho\mathbf{W}))^{-1}$$

For CAR model

$$\mathbf{C} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{M}$$

Where M is a matrix of conditional variances – if constant $M = I\sigma^2$

For Moving Average model

$$\mathbf{C} = \sigma^2(\mathbf{I} - \rho\mathbf{W})(\mathbf{I} - \rho\mathbf{W})^T$$

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Example

$$W = \begin{pmatrix} 0.0 & 0.3 & 0.7 \\ 0.2 & 0.0 & 0.8 \\ 0.7 & 0.3 & 0.0 \end{pmatrix} \qquad \rho = 0.5$$

SAR

$$\left((\mathbf{I} - \rho\mathbf{W})^T(\mathbf{I} - \rho\mathbf{W})\right)^{-1} = \begin{pmatrix} 1.8134 & 0.9652 & 1.3076 \\ 0.9652 & 1.6875 & 1.1513 \\ 1.3076 & 1.1513 & 1.8993 \end{pmatrix}$$

Stationarity is not a requirement

Covariances differ for similar spatial proximity values

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Fitting Models

Autocorrelated errors model could be fit to observed data if ρ is known

Then C could be estimated and a generalized least squares model could be fit on the model

$$\mathbf{Y} = \mathbf{X\beta} + \mathbf{U}$$

Typically ρ is not know and needs to be estimated along with β often using maximum likelihood

An alternative pragmatic approach is to guess a value for ρ

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Fitting Models

Setting ρ equal to 1

$$\mathbf{Y} = \mathbf{X\beta} + \rho\mathbf{WY} - \rho\mathbf{WX\beta} + \varepsilon \text{ becomes}$$

$$\mathbf{Y} = \mathbf{X\beta} + \mathbf{WY} - \mathbf{WX\beta} + \varepsilon$$

May be rewritten as

$$(\mathbf{I} - \mathbf{W})\mathbf{Y} = (\mathbf{I} - \mathbf{W})\mathbf{X\beta} + \varepsilon$$

Which converts to an OLS regression of $(\mathbf{I} - \mathbf{W})\mathbf{Y}$ on $(\mathbf{I} - \mathbf{W})\mathbf{X}$

Setting ρ equal to 1 is only sensible if there are significant second order effects

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Fitting Models

Yet another alternative to maximum likelihood estimation is:

Regressing $(\mathbf{I} - \rho\mathbf{W})\mathbf{Y}$ on $(\mathbf{I} - \rho\mathbf{W})\mathbf{X}$ by ordinary least squares for a sequence of different values of ρ

Choose the outcome with the most acceptable residuals

These are ad hoc fitting methods that do not provide standard errors and confidence intervals for ρ

To obtain confidence intervals requires maximum likelihood estimation

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Maximum Likelihood Estimation

For maximum likelihood estimation - must make a distributional assumption

Assume Y_i are normally distributed with mean $\mathbf{X\beta}$ and covariance matrix $\mathbf{C}$

Under these assumptions the log likelihood function for n observations on y is:

$$l() \propto -\log|\mathbf{C}| - (y - \mathbf{X\beta})^T \mathbf{C}^{-1}(y - \mathbf{X\beta})$$

Where $\mathbf{C} = \sigma^2(\mathbf{I} - \rho\mathbf{W})^T((\mathbf{I} - \rho\mathbf{W}))^{-1}$ For autocorrelated errors model

Log likelihood depends on parameters ρ, β, σ^2

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Maximum Likelihood Estimation Approach

Get an initial estimate of β using ordinary least squares on the model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \varepsilon$

Maximize the log likelihood l() with respect to σ^2 and ρ, assuming β take the values of their most recent estimates

Obtain new estimates of β by generalized least squares on the model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{U}$

With $C^{-1} = \hat{C}^{-1} = \frac{1}{\hat{\sigma}^2}\left((\mathbf{I} - \hat{\rho}\mathbf{W})^T(\mathbf{I} - \hat{\rho}\mathbf{W})\right)$

Iterate on steps 2 and 3 until appropriate convergence of parameter estimates β, σ^2, ρ

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Ordinary Least Squares Result for Log Heart Attack and Log Jarman Scores

Call: lm(formula = Dhapol3.field$Log.Heart ~ Dhapol3.field$LogJarman)

Residuals:

Min	1Q	Median	3Q	Max
-0.7561	-0.1963	-0.01644	0.1994	0.5076

Coefficients:

	Value	Std. Error	t value	Pr(>|t|)
(Intercept)	1.7175	0.4271	4.0210	0.0001
Dhapol3.field$LogJarman	0.6231	0.0930	6.6992	0.0000

Residual standard error: 0.2487 on 188 degrees of freedom

Multiple R-Squared: 0.1927

F-statistic: 44.88 on 1 and 188 degrees of freedom, the p-value is 2.361e-010

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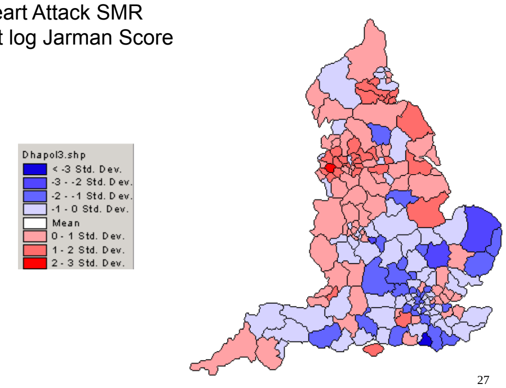
Residuals from OLS

Log Heart Attack SMR against log Jarman Score



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Moran's I for OLS residuals

Statistic = "moran" Sampling = "free"

Correlation = 0.4723

Variance = 0.001937

Std. Error = 0.04401

Normal statistic = 10.85

Normal p-value (2-sided) = 1.947e-27

Summary of the permutation-correlations :

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.1555	-0.03417	-0.006246	-0.006133	0.0215	0.134

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REGRESSION

Data set : **DHApol**

Dependent Variable : **LOGMYCARD Number of Observations: 190**

Mean dependent var : 1.98753 Number of Variables : 3

S.D. dependent var : 0.119557 Degrees of Freedom : 187

R-squared	: 0.432473	F-statistic	: 71.2499
Adjusted R-squared	: 0.426403	Prob(F-statistic)	: 9.9481e-024
Sum squared residual:	1.5413	Log likelihood	: 187.769
Sigma-square	: 0.00824225	Akaike info criterion	: -369.539
S.E. of regression	: 0.0907868	Schwarz criterion	: -359.798
Sigma-square ML	: 0.00811211		
S.E of regression ML:	0.0900672		

Variable	**Coefficient**	**Std.Error**	**t-Statistic**	**Probability**
CONSTANT	1.08518	0.1605553	6.758919	0.0000000
LOGJARMAN	0.39226	0.0823932	4.760911	0.0000039
Y	4.971268e-006	5.593223e-007	8.888021	0.0000000

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REGRESSION DIAGNOSTICS

MULTICOLLINEARITY CONDITION NUMBER 58.91717

(Extreme Multicollinearity)

TEST ON NORMALITY OF ERRORS

TEST	DF	VALUE	PROB
Jarque-Bera	2	1.233371	0.5397305

DIAGNOSTICS FOR HETEROSKEDASTICITY

RANDOM COEFFICIENTS

TEST	DF	VALUE	PROB
Breusch-Pagan test	2	0.2038841	0.9030819
Koenker-Bassett test	2	0.2129618	0.8989922

SPECIFICATION ROBUST TEST

TEST	DF	VALUE	PROB
White	5	2.610991	0.7596946

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DIAGNOSTICS FOR SPATIAL DEPENDENCE

FOR WEIGHT MATRIX : **dhad.GAL (row-standardized weights)**

TEST	**MI/DF**	**VALUE**	**PROB**
Moran's I (error)	0.323742	7.1251296	0.0000000
Lagrange Multiplier (lag)	1	0.5400184	0.4624251
Robust LM (lag)	1	4.6905360	0.0303292
Lagrange Multiplier (error)	1	44.9998215	0.0000000
Robust LM (error)	1	49.1503391	0.0000000
Lagrange Multiplier (SARMA)	2	49.6903575	0.0000000

Robust LM error - Checks for error dependence in possible presence of missing lagged dependent variable

Robust LM lag - Checks for lag dependence in possible presence of missing lagged error

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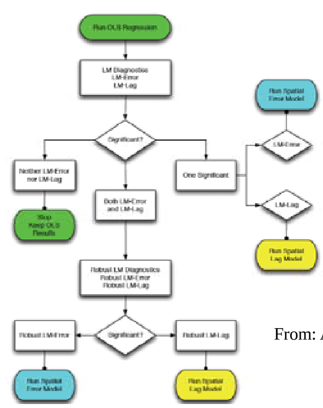

From: Anselin 2005

Spatial regression decision process

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REGRESSION
SUMMARY OF OUTPUT: SPATIAL ERROR MODEL - MAXIMUM LIKELIHOOD ESTIMATION

Data set : **DHApol**
Spatial Weight : **dhad.GAL**
Dependent Variable : **LOGMYCARD Number of Observations: 190**
Mean dependent var : 1.987531 Number of Variables : 3
S.D. dependent var : 0.119557 Degree of Freedom : 187
Lag coeff. (Lambda) : 0.735180

R-squared : 0.600139 R-squared (BUSE) : -
Sq. Correlation : - Log likelihood : 211.802500
Sigma-square : 0.005716 Akaike info criterion : -417.605
S.E of regression : 0.0756011 Schwarz criterion : -407.863928

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Variable	Coefficient	Std.Error	z-value	Probability
CONSTANT	1.524308	0.1563497	9.74935	0.0000000
LOGJARMAN	0.1943649	0.07773776	2.500264	0.0124101
Y	3.835949e-006	1.498925e-006	2.559134	0.0104934
LAMBDA	0.7351797	0.05527206	13.30111	0.0000000

REGRESSION DIAGNOSTICS
DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS

TEST	DF	VALUE	PROB
Breusch-Pagan test	2	0.2311871	0.8908372

DIAGNOSTICS FOR SPATIAL DEPENDENCE
SPATIAL ERROR DEPENDENCE FOR WEIGHT MATRIX : **dhad.GAL**

TEST	DF	VALUE	PROB
Likelihood Ratio Test	1	48.06611	0.0000000

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Spatial Linear Regression Results – SAR model

```
slm(formula = Dhapol3.field$Log.heart ~ Dhapol3.field$Logjarman,
cov.family = SAR, spatial.arglist = list(neighbor = Dhapol3.neighbor))
Residuals:
    Min      1Q    Median     3Q      Max
 -0.6736  -0.1391  0.01342  0.1167  0.371
Coefficients:
                          Value Std. Error t value  Pr(>|t|)
          (Intercept)     3.5692 0.3838     9.2994  0.0000
Dhapol3.field$Logjarman   0.2249 0.0826     2.7233  0.0071
Residual standard error: 0.185864 on 187 degrees of freedom

rho =  0.1297
Iterations =  7
Gradient norm =  1.745e-5
Log-likelihood =  -189.6
```

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REGRESSION: SPATIAL LAG MODEL - MAXIMUM LIKELIHOOD ESTIMATION

Data set : **DHApols**
Spatial Weight : **dhad.GAL**
Dependent Variable : **LOGMYCARD Number of Observations: 190**
Mean dependent var : 1.98753 Number of Variables : 4
S.D. dependent var : 0.119557 Degrees of Freedom : 186
Lag coeff. (Rho) : 0.0321168

R-squared : 0.434120 Log likelihood : 188.031
Sq. Correlation : - Akaike info criterion : -368.063
Sigma-square : 0.00808857 Schwarz criterion : -355.075
S.E of regression : 0.0899365

Variable	Coefficient	Std.Error	z-value	Probability
W_LOGMYCARD	0.0321168	0.04401245	0.729721	0.4655606
CONSTANT	1.050016	0.1697981	6.183907	0.0000000
LOGJARMAN	0.3805263	0.08228289	4.62461	0.0000038
Y	4.772283e-006	6.204649e-007	7.691464	0.0000000

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REGRESSION DIAGNOSTICS
DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS

TEST	DF	VALUE	PROB
Breusch-Pagan test	2	0.4173125	0.8116742

DIAGNOSTICS FOR SPATIAL DEPENDENCE
SPATIAL LAG DEPENDENCE FOR WEIGHT MATRIX : **dhad.GAL**

TEST	DF	VALUE	PROB
Likelihood Ratio Test	1	0.5239573	0.4691585

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OLS

Variable	Coefficient	Std.Error	t-Statistic	Probability
CONSTANT	1.08518	0.1605553	6.758919	0.0000000
LOGJARMAN	0.39226	0.0823932	4.760911	0.0000039
Y	4.971268e-006	5.593223e-007	8.888021	0.0000000

Spatial Lag model

Variable	Coefficient	Std.Error	z-value	Probability
W_LOGMYCARD	0.0321168	0.04401245	0.729721	0.4655606
CONSTANT	1.050016	0.1697981	6.183907	0.0000000
LOGJARMAN	0.3805263	0.08228289	4.62461	0.0000038
Y	4.772283e-006	6.204649e-007	7.691464	0.0000000

Spatial error model

Variable	Coefficient	Std.Error	z-value	Probability
CONSTANT	1.524308	0.1563497	9.74935	0.0000000
LOGJARMAN	0.1943649	0.07773776	2.500264	0.0124101
Y	3.835949e-006	1.498925e-006	2.559134	0.0104934
LAMBDA	0.7351797	0.05527206	13.30111	0.0000000

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Comparison of OLS and SAR

OLS

Coefficients:

	Value	Std. Error	t value	Pr(>|t|)
(Intercept)	1.7175	0.4271	4.0210	0.0001
Dhapol3.field$LogJarman	**0.6231**	**0.0930**	**6.6992**	**0.0000**

SAR

Coefficients:

	Value	Std. Error	t value	Pr(>|t|)
(Intercept)	3.5692	0.3838	9.2994	0.0000
Dhapol3.field$LogJarman	**0.2249**	**0.0826**	**2.7233**	**0.0071**

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Spatial Linear Regression Results – SAR model vs OLS

SAR

Spatial Correlation Estimate
Statistic = "moran" Sampling = "free"
Correlation = -0.0488
Variance = 0.001937
Std. Error = 0.04401
Normal statistic = -0.9887
Normal p-value (2-sided) = 0.3228
Summary of the permutation-correlations :
Min. 1st Qu. Median Mean 3rd Qu. Max.
-0.1562 -0.0353 -0.0050 -0.00531 0.0229 0.1821
permutation p-value = 0.842

OLS

Spatial Correlation Estimate
Statistic = "moran" Sampling = "free"
Correlation = 0.4723
Variance = 0.001937
Std. Error = 0.04401
Normal statistic = 10.85
Normal p-value (2-sided) = 1.944e-27
Summary of the permutation-correlations :
Min. 1st Qu. Median Mean 3rd Qu. Max.
-0.1348 -0.03701 -0.00701 -0.005467 0.02514 0.1751
permutation p-value = 0

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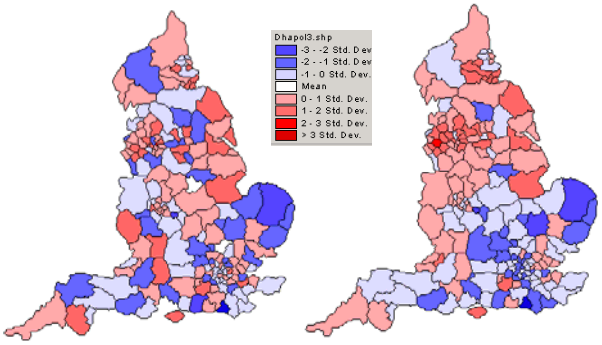
Residuals from SAR model vs OLS



SAR **OLS**

Example – Breeding Bird Habitat Models

Comparison of

1. OLS environment model

Fit ordinary least squares to several habitat variables

Habitat variables with P>0.01 were retained

2. OLS trend/ environment model

$$z = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2 + b_6 x^3 + b_7 x^2 y + b_8 xy^2 + b_9 y^3$$

Where z is square root transformed species counts

3. OLS trend/ environment + Autocorrelation (CAR) model

Source: Lichstein et al. 2002 Ecological Monographs

Example – Breeding Bird Habitat Models

TABLE 2. Ordinary least squares (OLS) and conditional autoregressive (CAR) models of bird abundance.

	R^2						Neighborhood effect	
Species	OLS env	trend	OLS trend/ env	autocor	CAR trend/ env	$\hat{\rho}$	200 m†	400 m‡
Chestnut-sided Warbler	0.48	0.26	0.53	0.17	0.55	3361.6	0.25	0.06
Hooded Warbler	0.20	0.08	0.22	0.12	0.25	17.3	0.26	0.13
Black-throated Blue Warbler	0.36	0.22	0.39	0.26	0.46	19.0	0.29	0.14

Notes: R^2 values are given for the following models: "OLS env" = habitat only; "trend" = broadscale spatial trend surface; "OLS trend/env" = habitat + trend; "autocor" = fine-scale autocorrelation (conditional autoregressive [CAR] model with only intercept and ρ); "CAR trend/env" = habitat + trend + autocorrelation. In CAR models, neighborhood effects are modeled by the estimated spatial parameter, $\hat{\rho}$, along with the neighbor weights: w_{ij} = (1/distance$_{ij}$)^2 for the Chestnut-sided Warbler and 1/distance$_{ij}$ for the Hooded and the Black-throated Blue Warblers. "Neighborhood effect" is the expected increase in the response (square-root-transformed count) at location i where the sum of the mean-centered responses at locations j in i's spatial neighborhood is +3. Neighborhood effect was calculated as $\hat{\rho}\Sigma_{j\neq i} w_{ij}(Y_j - \mu_j)$, which is the autoregressive component of the conditional expectation of Y_i in the CAR model. As an arbitrary but realistic example, we assumed that $\Sigma(Y_j - \mu_j) = 3$ for locations j in i's spatial neighborhood.

† Neighborhood effect was calculated assuming all j in i's spatial neighborhood are 200 m away from i.

‡ Neighborhood effect was calculated assuming all j in i's spatial neighborhood are 400 m away from i.